The Sundial Primer - "Check-A-Dial"

created by
Carl Sabanski

General

Sunny day! I am happy to see that you are interested in obtaining a functional sundial for your yard or garden. You will discover that this task will take some effort on your part. In the end, when your sundial is finally in the place that is now only in your mind's eye, it will have been a very satisfying journey. You will discover many novelty and "wanna-be" sundials where it is not even worth the effort to read the instructions that come with them. Why? Because they will not tell you the time no matter where or how you position them. There are more of these then you can imagine. Why? Because people buy them under some misconception that they will work. And some of these sundials are expensive!

To find a sundial that will actually tell time with some accuracy you must first be willing to spend some time learning about how a sundial works and what kinds of time they can indicate. With the information presented here you can then evaluate a sundial to determine if it will actually function. Then you need to know how to change this time to clock time, which is what we all use every day.

Timekeeping is based upon the period in which the earth makes one complete revolution upon its axis. This period is called a day, which is divided into 24 hours, each hour having 60 minutes and each minute 60 seconds. This period of time, the day, is complete when the sun starts and returns to the same point on the earth. As the earth rotates 360° in a 24-hour period:

1 hour = 15° or 1° = 4 minutes

Hour Angle (h, HA): the angle corresponding to the sun's position around its daily (apparent) orbit. Measured westward from local noon, it increases at a rate of 15° per hour.

A particular geographical location on earth is described by its latitude and longitude.

Latitude (ø): is the angular position of a place on the Earth's surface measured north or south of the equator. Positive values in the Northern hemisphere, negative in the Southern.

Longitude (λ): is the angular location of a place on the Earth's surface measured east or west of the Prime meridian through Greenwich. Longitudes west are positive and east are negative.

As the earth rotates and the sun appears to move, it will be over only one particular longitude at a given time. As an observer, this particular longitude, which passes through your location is called a meridian. If you are using the sun to tell the time, no two observers will share the same time unless they are on the same meridian.

Local Apparent Time (L.A.T.): this is solar time, as derived from the real sun at any particular location. This is the kind of time that is shown on most sundials.

Telling time by this method is very inconvenient. In a large city there could be a significant time difference between the east and west ends of the city. How would you ever make it to a meeting on time?! There is a second problem with this method of telling time. This is the fact that measuring time using the sun results in days of varying lengths during the year.

Instead of the real sun, we use an imaginary mean sun that moves at a constant speed equal to the average speed of the real sun. Divide this day into twenty-four equal parts and you have:

Mean Solar Time: a measure of day based conceptually on the diurnal (daily) motion of the mean fictitious sun, under the assumption that the earth's rate of rotation is constant.

If you start the real sun and the mean sun at a time when they coincide, over the year, the mean sun will sometimes lag or lead the real sun. At the end of the year, though, they will meet again. Like the real or apparent sun, the mean sun is over only one particular meridian at a given time. Mean solar time is still localized.

Local Mean Time (LMT): this is solar time which has been corrected for the Equation of Time but not for longitude. The difference between the Local Mean Time and the Local Apparent time is known as the Equation of Time.

Equation of Time (E, EoT): the time difference between Local Apparent Time (apparent solar time) and Mean Solar Time at the same location. Its value varies between extremes of about +14 minutes in February and -16 minutes in October. It arises because of the elliptical orbit of the earth, and the tilt of the earth's axis to the ecliptic. The preferred usage by diallists is:

Mean Solar Time = Local Apparent (Solar) Time + EoT

but this convention is by no means universal and the opposite sign is used in modern almanacs. Irrespective of the sign convention adopted, the sundial will appear slow compared to the mean time in February, and fast in October/November. EoT varies continuously, but is usually tabulated for noon each day at a particular location.

Figure 1 is a graph of the Equation of Time.

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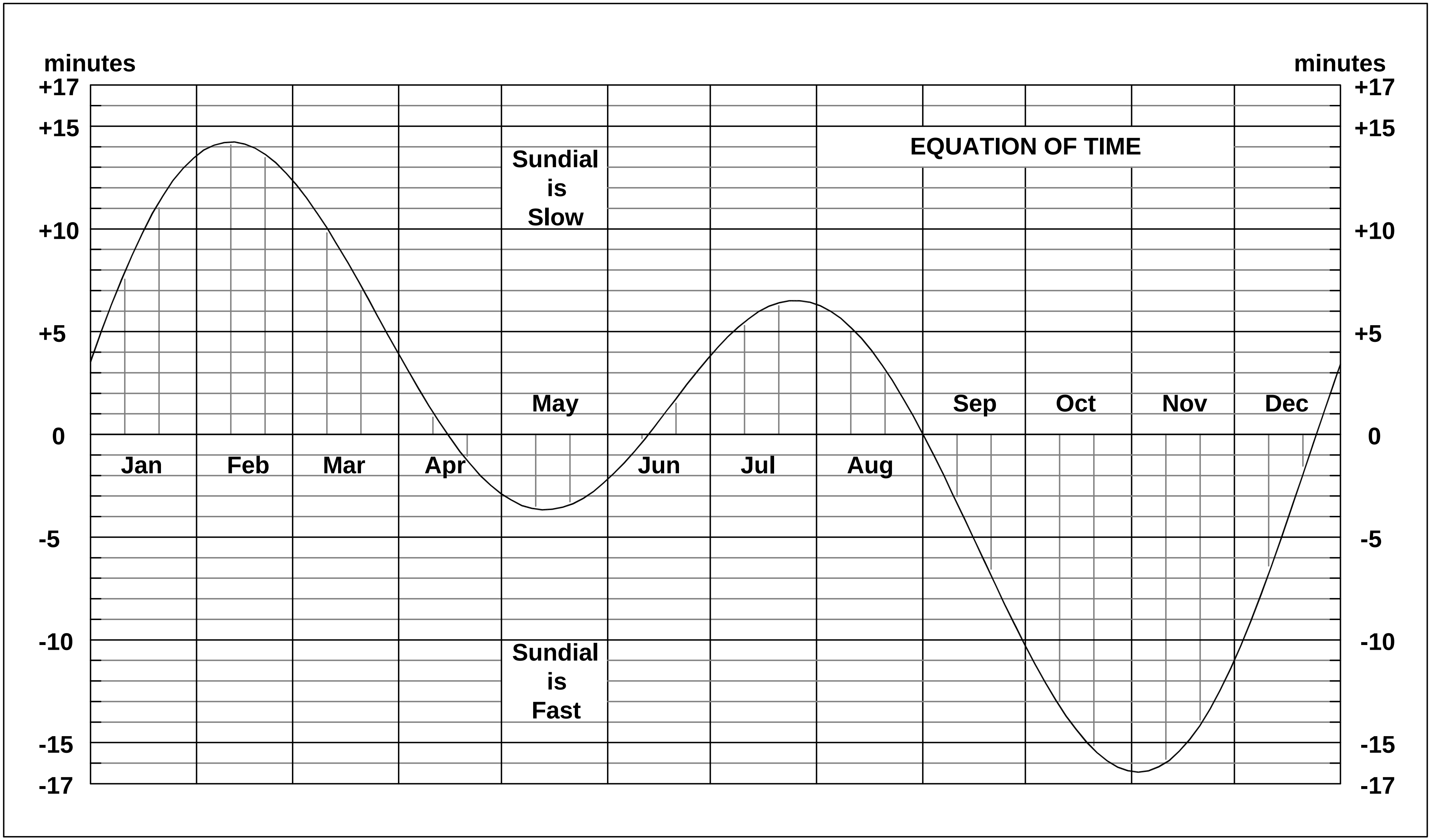
Figure 1



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Note that Mean Solar Time and the Apparent Solar Time coincide four times a year.

By applying the Equation of Time to a sundial that indicates Local Apparent Time, it is possible to obtain the Local Mean Time of the place where the dial is located. This correction can be made by using an EoT graph as shown above and adding or subtracting the appropriate number of minutes from the sundial reading. There are also sundials with specially designed gnomons that will account for the EoT.

Clocks, however, indicate standard time.

Standard Time (ST): is mean solar time at the central meridian of a given time zone.

Or as a further definition civil time.

Civil (Clock) Time: the legally accepted time scale in a particular country or region. It is based on the standard time for that standard time zone, but may have fixed differences (e.g. BST/DST). Measured in modern hours from the most recent midnight, with either a 24 hour or 2 x 12 hour format.

Standard Time Zone (TZ): a geographical region which uses the same civil (clock) time. These are approximately regions between two lines of longitude, set 15° apart, and hence 1 hour time between adjacent zones. For the UK, which is in Zone 0, the standard meridian is the Prime Meridian at Greenwich, and the zone extends nominally 7½° west to 7½° east.

Within a given time zone, a sundial located at the central meridian of that time zone will indicate clock time with the appropriate application of the EoT. If the sundial is located to the east, it will read fast and if it is located to the west it will read slow. As every 1° movement of the sun is equivalent to 4 minutes of time, a correction for longitude can be applied to a sundial that indicates Local Apparent Time as follows:

Longitude Correction = (Central Meridian - Local Meridian) * 4 minutes

The longitude correction can be applied manually to the sundial indication or it can also be incorporated into the graph of the Equation of Time. The correction can also be incorporated into the design of the hour line angles of the sundial which would then indicate Zonal Solar Time. This type of sundial will not be discussed any further.

Zonal Solar Time: denotes solar time at a time zone meridian. Thus it is Local Apparent Time with a longitude correction but without EoT.

One last correction that can be made is for:

Daylight Saving Time (DST): civil time during the summer in much of the USA (and some other countries) obtained by advancing clock time by one hour from local standard time.

or

British Summer Time (BST): civil time in he UK during the summer, one hour ahead of GMT (Greenwich Mean Time).

or

Summer Time: a generic term for BST, DST, etc.

This can be done by having two sets of numbers for the hour lines or in locations where the dial is covered with snow for a long period of time, you may choose to number it to read DST. Some sundials have an adjustment for DST.

Now, to correct the time indicated by a sundial designed to read Local Apparent or Solar Time and obtain Standard or Clock Time:

Standard Time = Local Apparent Time + Longitude Correction + Equation of Time + Daylight Saving Time

where:

Longitude Correction: positive west of the central meridian; negative east of the central meridian
Equation of Time: positive when the dial is slow; negative when the dial is fast
Daylight Saving Time: 1 or 0

It should be noted that there may also be variations in the standard time due to local legislated time zone variations. Be sure to check for this.

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General

The proceeding information has hopefully helped you to understand what kinds of time are of interest to someone using a sundial, what kinds of time a sundial can show and how to correct a sundial time reading to one we are familiar with, clock time.

Figure 2 is provided to help you with task of correcting a solar time sundial reading to clock time. What you will be doing is combining the longitude and Equation of Time corrections into one chart. You will need to add or subtract only one number from the time shown by the sundial to obtain clock time. To complete the simple calculations required to fill in the blanks you need to know the longitude where the sundial is located and the longitude of the local time zone meridian. The following examples show how to perform the calculations for various longitudes. It is important to remember that west longitudes are positive and east longitudes are negative.

Example 1:

a) Longitude of Sundial: +95° W
b) Longitude of Local Time Zone Meridian: +90° W
c) Longitude Difference: a - b = 95 - 90 = 5° The sundial is located west of the local time zone meridian. The sundial is slow.

If the value of "c" is positive the sundial is located west of the local time zone meridian and its time indication is slow. If the value of "c" is positive the sundial is located east of the local time zone meridian and its time indication is fast. tom made.

d) Longitude Correction: c x 4 = 5 x 4 = 20 minutes
e) The values that would be entered into the blanks on the left side of Figure 2 would be the values of the EoT on the right plus 20. From the top to bottom the nine values are: +37, +35, +30, +25, +20, +15, +10, +5 and +3. All the values are positive and you would always be adding a value to the sundial reading.

Example 2:

a) Longitude of Sundial: -95° E
b) Longitude of Local Time Zone Meridian: -90° E
c) Longitude Difference: a - b = -95 - (-90) = -95 + 90 = -5° The sundial is located east of the local time zone meridian. The sundial is fast.
d) Longitude Correction: c x 4 = -5 x 4 = -20 minutes
e) The values that would be entered into the blanks on the left side of Figure 2 would be the values of the EoT on the right plus -20. Adding a negative number is the same as subtracting the number. From the top to bottom the nine values are: -3, -5, -10, -15, -20, -25, -30, -35 and -37. All the values are negative and you would always be subtracting a value from the sundial reading.

Example 3:

a) Longitude of Sundial: 87° W
b) Longitude of Local Time Zone Meridian: 90° W
c) Longitude Difference: a - b = 87 - 90 = -3° The sundial is located east of the local time zone meridian. The sundial is fast.
d) Longitude Correction: c x 4 = -3 x 4 = -12 minutes
e) The values that would be entered into the blanks on the left side of Figure 2 would be the values of the EoT on the right plus -12. From the top to bottom the nine values are: +5, +3, -2, -7, -12, -17, -22, -27 and -29. The values are both positive and negative and you would add or subtract a value from the sundial reading.

Example 4:

a) Longitude of Sundial: -87° E
b) Longitude of Local Time Zone Meridian: -90° E
c) Longitude Difference: a - b = -87 - (-90) = -87 + 90 = 3° The sundial is located west of the local time zone meridian. The sundial is slow.
d) Longitude Correction: c x 4 = 3 x 4 = 12 minutes
e) The values that would be entered into the blanks on the left side of Figure 2 would be the values of the EoT on the right plus 20. From the top to bottom the nine values are: +29, +27, +22, +17, +12, +7, +2, -3 and -5. The values are both positive and negative and you would add or subtract a value from the sundial reading.

This calculation can be performed for any sundial location and if you have any problems ask a friend to give you some help.

Now let's go on to the next phase and learn what you need to look for when you are evaluating a sundial. Evaluation criteria will be provided for three of the most common sundial that are commercially supplied; the horizontal sundial, the vertical sundial (direct south vertical in the Northern Hemisphere and direct north in the Southern Hemisphere) and the equatorial sundial (equatorial ring or armillary).

What you need to know is the location of your sundial, the latitude and longitude. The following instructions pertain to sundials that indictae local apparent or solar time. You will use your latitude to check the sundial and longitude to calculate the longitude correction. Commercial sundials are usually made for only one latitude and it may not be possible to obtain one for your specific latitude unless it is custom made. However, a properly made sundial for one latitude can work at another latitude by tilting it. This option will discussed for each sundial.

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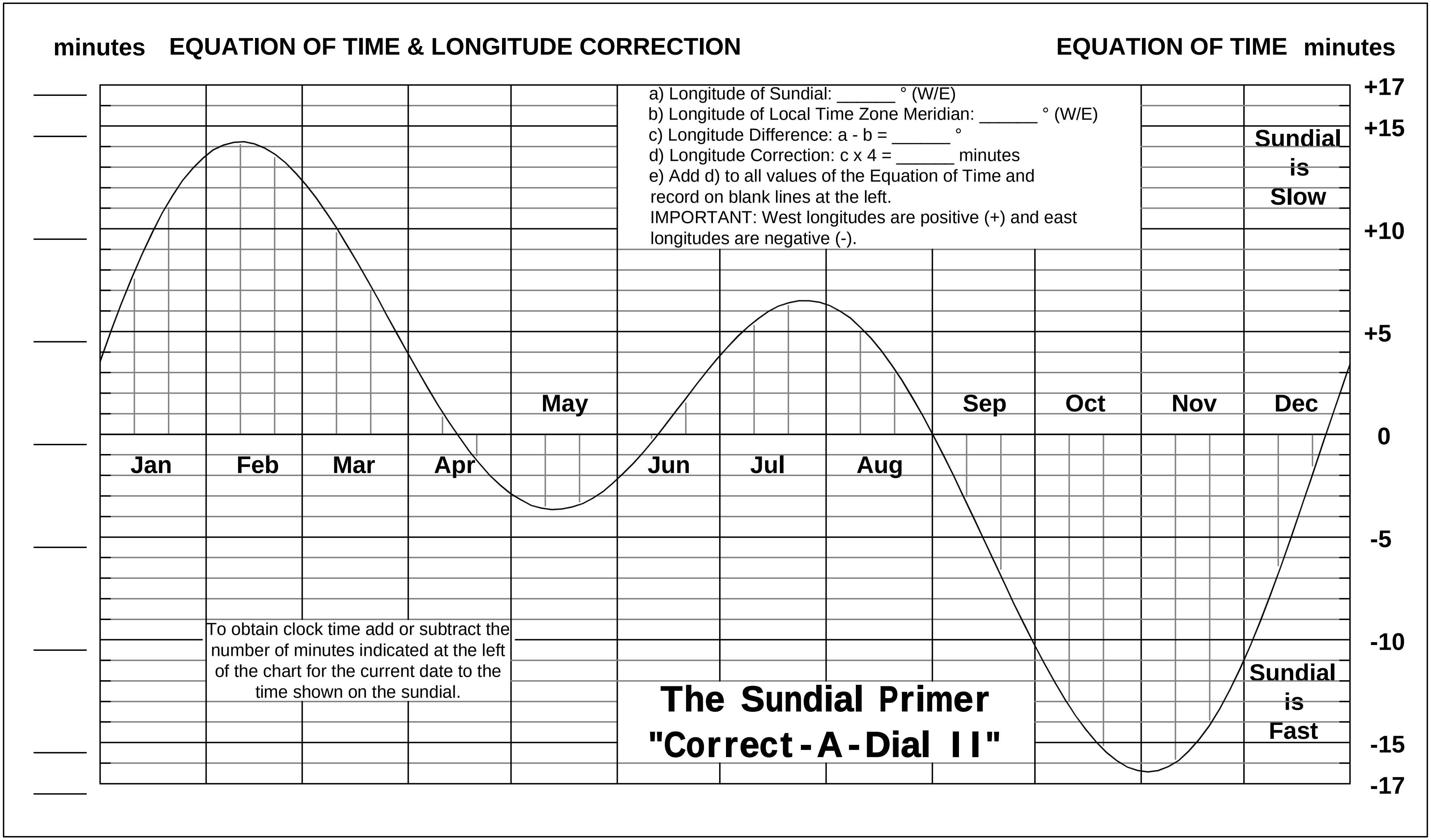
Figure 2

Complete this chart for your sundial's longitude and cut it out for easy use. You can laminate the chart to protect it. Completer versions of this chart are available at Sunny Day U. There are 61 models available.

This chart will help you convert the local apparent or solar time shown by your sundial to clock time. Examples are given in the "General" section of "Check-A-Dial".



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General

Sundial: an instrument for telling the time and/or date from the position of the Sun.

As the sundial is an instrument that monitors and translates the sun's position to indicate time it must follow certain rules. These rules are dictated by the movement of the sun during any given day of the year. By understanding the relationship between the sun and the earth various types of sundials can be designed. Some are relatively simple and others quite complex. You do not need to know how to design a sundial to find one that will indicate the time correctly. You do need to evaluate the sundial in terms of the information provided here. If the sundial meets the criteria presented it will function correctly. If it does not it will not be able to tell the time accurately no matter what you do to it.

Before discussing the horizontal, vertical and equatorial sundials individually there is a device that will help you when you are shopping for a sundial. This device is the "Check-A-Dial". There are four of these devices available. Two are used for the horizontal and equatorial sundials and two are for the vertical and equatorial sundials. With the "Check-A-Dial" you will be able to check the following:

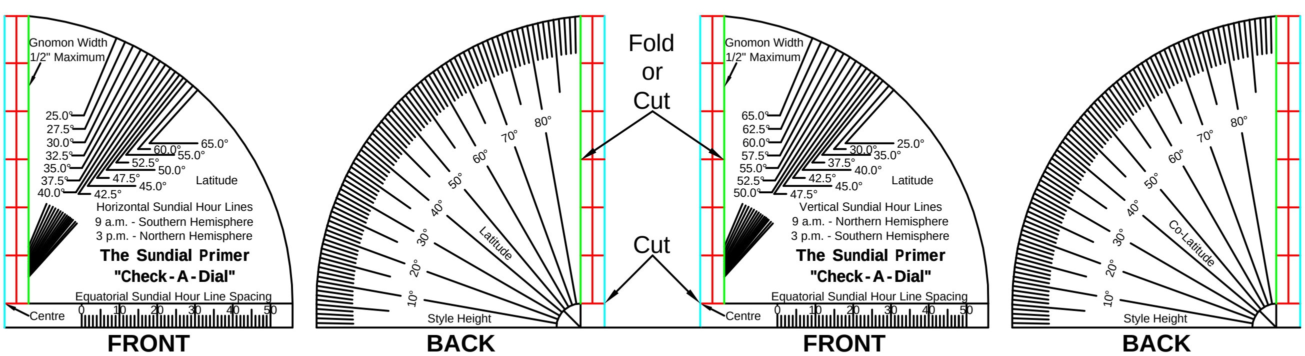
1. Horizontal Sundial Hour Lines: 9 a.m. Northern Hemisphere and 3 p.m. Southern Hemisphere
2. Horizontal Sundial Hour Lines: 9 a.m. Southern Hemisphere and 3 p.m. Northern Hemisphere
3. Vertical Sundial Hour Lines: 9 a.m. Northern Hemisphere and 3 p.m. Southern Hemisphere
4. Vertical Sundial Hour Lines: 9 a.m. Southern Hemisphere and 3 p.m. Northern Hemisphere
5. Horizontal Sundial Style Height: equal to the latitude of the sundial's location
6. Vertical Sundial Style Height: equal to the co-latitude (90° - latitude) of the sundial's location
7. Equatorial Sundial Hour Line Spacing

It might be handy to have both for a given type of sundial. Print the "Check-A-Dial" on card stock and cut it out. Fold it in half along the centre line (cyan) and glue the two halves together. Laminate it in plastic to make it more rugged. See Figure 3 for details.

The "Check-A-Dial" is designed to be aligned with the origin and noon line of a solar time horizontal or vertical sundial. It will accommodate gnomons up to 1/2 inch wide. To do this you can either cut along the short red line at the origin and fold along the green line or cut out the entire piece. If the gnomon is slightly wide cut a little further beyond the red line. The use of the "Check-A-Dial" will be discussed for each type of sundial under the appropriate section.

The purpose of the "Check-A-Dial" is to give you the ability to make a couple of basic measurements and use these to determine whether the sundial will indicate the time correctly. Many sundials will not even make it to the "Check-A-Dial" stage as you will soon see. There are a few other visual checks that will first be made that may result in the sundial being rejected.

None of this is difficult but it does take some time. As you evaluate more sundials it will become very easy to determine whether or not you should move on to another sundial. So get your "Check-A-Dial" devices ready and let's go!



Horizontal Sundial "Check-A-Dial" **Vertical Sundial "Check-A-Dial"**

Figure 3

Your sundial evaluation kit should include the following items:

1. The appropriate "Check-A-Dial".
2. A 6-inch clear plastic ruler.
3. A length of string 18 to 24 inches long.
4. Pencil and paper for making notes.

That's it!

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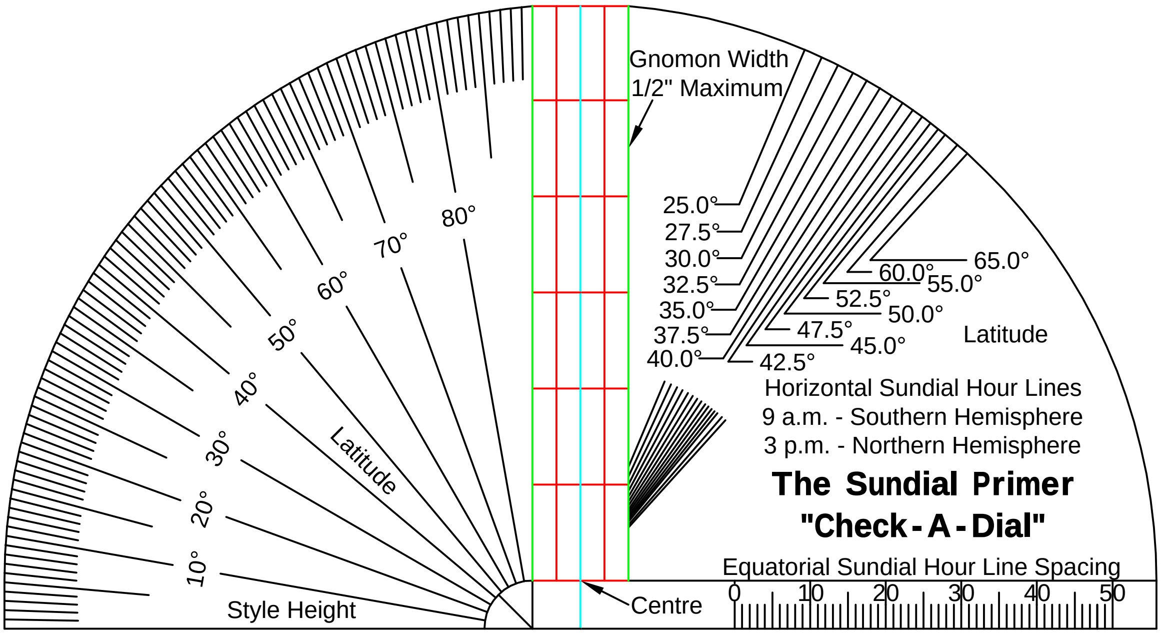

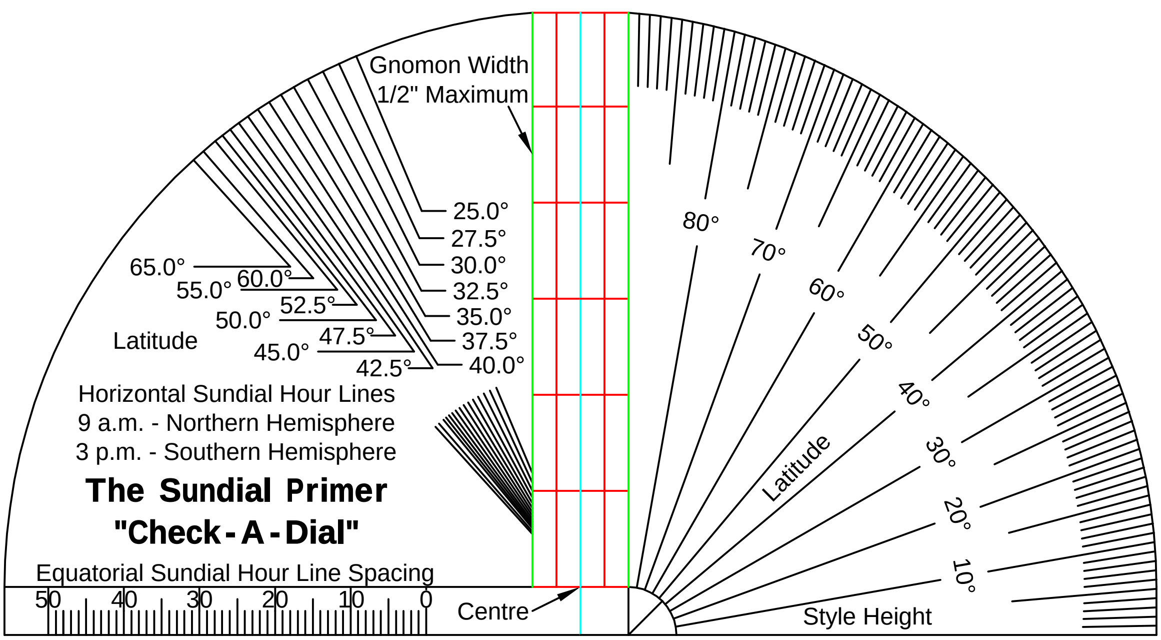

Print the "Check-A-Dial" on card stock and cut it out. Fold on the centre line (cyan) and glue the two halves together. Laminate the "Check-A-Dial" for protection.

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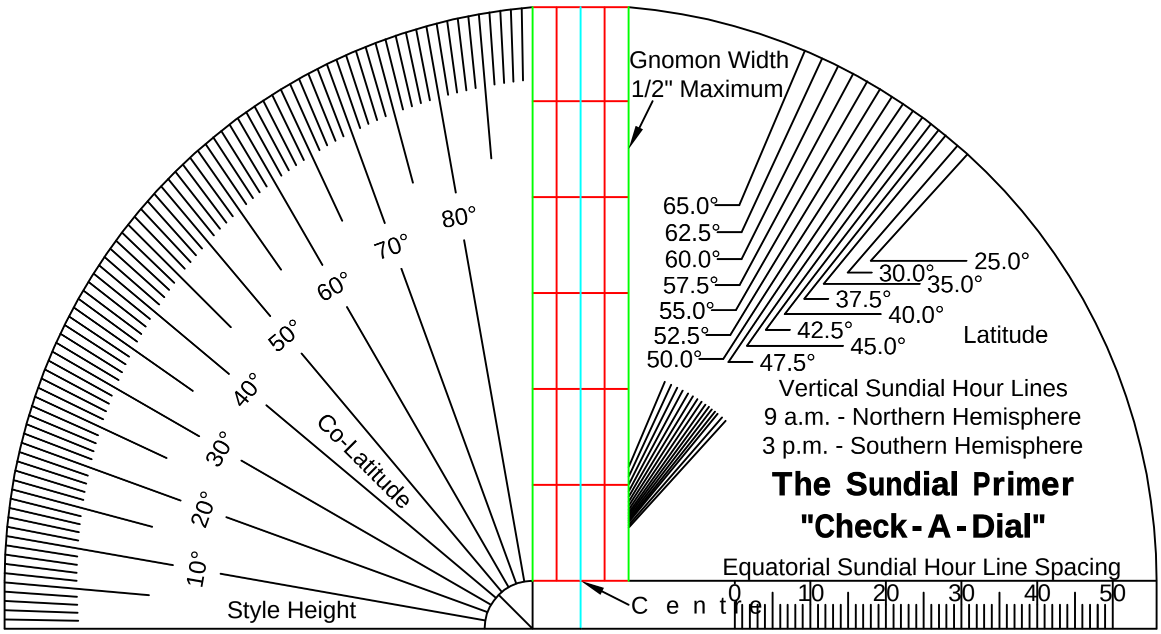

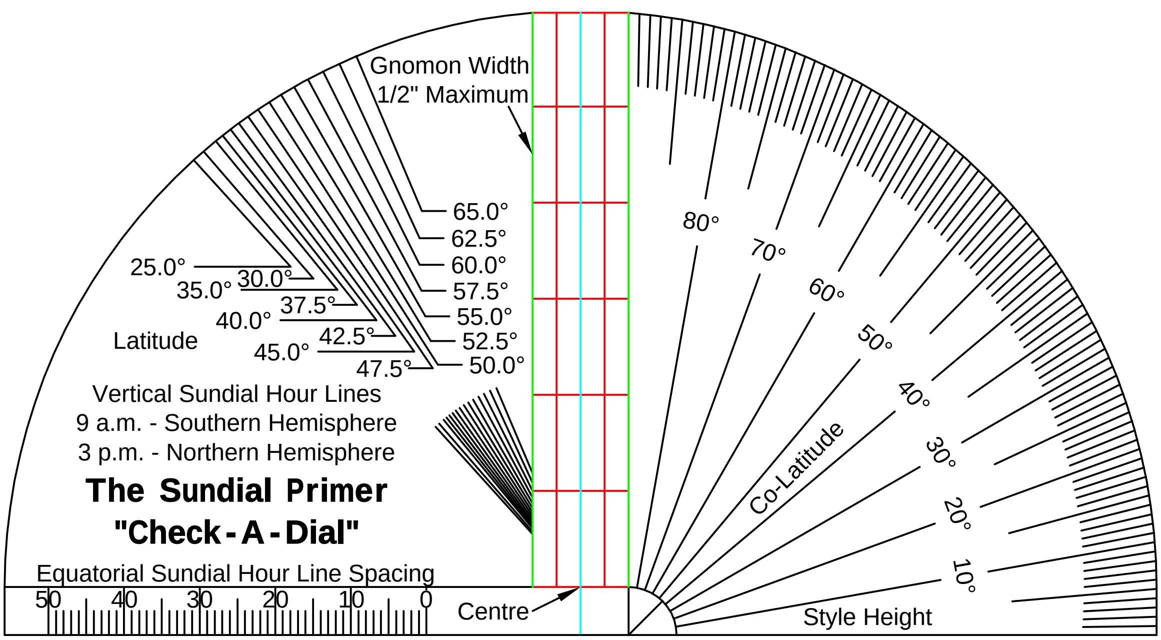

Print the "Check-A-Dial" on card stock and cut it out. Fold on the centre line (cyan) and glue the two halves together. Laminate the "Check-A-Dial" for protection.

The Sundial Primer - "Check-A-Dial"
Horizontal Sundial

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The first sundial to be discussed is likely to be the one most people are familiar with , the horizontal sundial.

Horizontal Sundial: the common or garden sundial with a horizontal dial plate and polar-pointing gnomon.

The following are some definitions that you should be familiar with. They apply to both horizontal and vertical sundials. Figure 4 illustrates these terms and will help to visualize them.

Dial Plate (Face): the physical plate on which the hour lines and furniture lie. It (usually) supports the gnomon.

Gnomon: the physical structure of a sundial which casts the shadow.

Hour Line: the line on a dial plate indicating the shadow position at a particular hour (includes fractional as well as whole hours).

Centre (of a dial): the point where all the hour lines, and a polar-pointing style, meet. This point does not always exist (e.g. on a polar dial or direct East or West dials, the lines meet at infinity). In simple horizontal and vertical dials, this point coincides with the root of a (thin) gnomon. In the case of a thick gnomon having two styles, there are two centres to the dial. The centre is often, but not always, the origin of the co-ordinate system used to describe the dial.

Noon Line (on a dial): simply the hour line corresponding to noon, it is the most important line from which the others are calculated.

Noon Gap: the gap on the hour scale of a dial to account for the finite thickness of the gnomon.

Style: the line in space which generates the shadow edge used to indicate the time on the dial plate. Note that a gnomon with finite thickness will have two styles (one along each of the upper edges) which will each be operational for parts of every day. If the gnomon is the form of a long rod, the style will be the virtual line running along the centre of the rod and the dial is read by estimating the centre of the shadow.

Style Height / Style Angle (SH): of a polar style is the angle that the style makes with the sub-style line.

Sub-Style: the line lying in the dial plane which is perpendicularly below (or behind for a vertical dial) the style.

Hour Line Angle (X, HLA): the angle that an hour line on a dial plate makes with the noon line. For a horizontal dial, the angle increases clockwise.

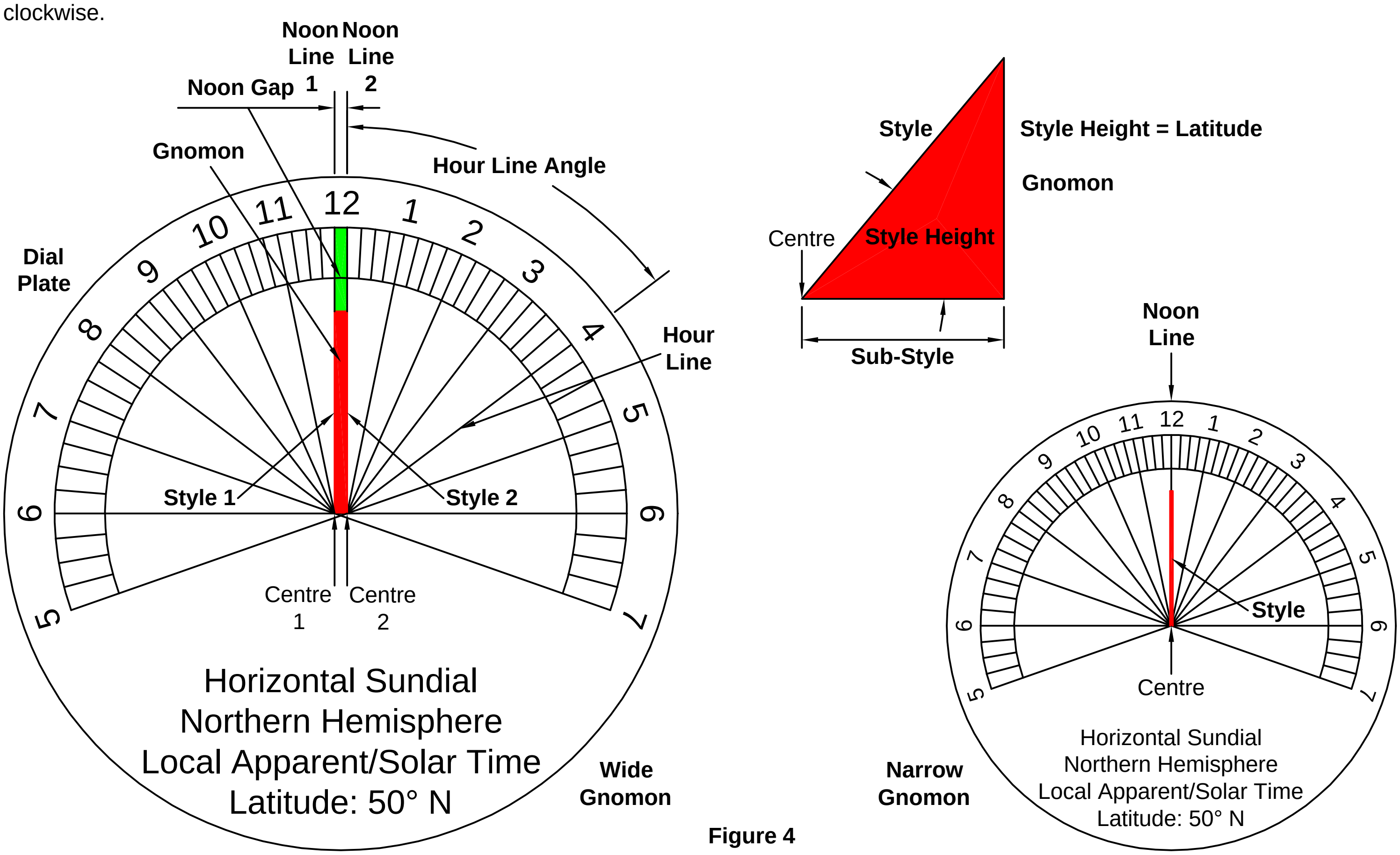

Figure 4

Horizontal Sundial

To assist with the following description a number of illustrations have been provided. Figure 5 illustrates a horizontal sundial for the Northern Hemisphere and Figure 6 for the Southern Hemisphere. Figures 7, 8, 9 and 10 illustrate a number of sundial layouts for the Northern and Southern Hemispheres. There are horizontal sundials shown designed for four different latitudes and with narrow and wide gnomons. Refer to these figures as well as Figure 4 to help you understand what is being discussed.

The following are important details to consider when you are evaluating a solar time horizontal sundial that you are considering to purchase.

1. The upper and lower surfaces of the dial plate should flat. Variations in the upper surface, particularly where the hour lines are located, could have an affect on the reading. If there are any raised design features they located below the centre of the sundial they should not be very high. Although they may look nice they could obscure the gnomon's shadow at times. Variations in the lower surface could make it difficult to install the dial plate level on its base.

2. Check that gnomon is securely fastened to the dial plate. The gnomon begins at the centre of the sundial and rises towards the noon (12) marker. All gnomons will have some width and this width must not change along the entire length of the sloped surface where the style(s) is located. The two edges (styles) of the sloped surface must be parallel, clean and sharp in order to cast a good shadow. The width of the gnomon must not increase below the sloped surface other than at the base where it may increase slightly to allow fastening to the dial plate. If wide features are incorporated into the gnomon they will obscure the gnomon's shadow. The gnomon must be straight and not tilted to the left or right.

3. The numbering of the hour lines must be clockwise for the Northern Hemisphere and counter clockwise for the Southern Hemisphere.

4. At this point the centre(s) of the sundial should be found based upon the position of the gnomon. Examine the hour lines near noon to determine whether the sundial design compensates for a thick gnomon. There will be two noon lines with a full hour before and after these lines. See Figures 8 and 10. Check this carefully. In most cases the sundial is likely to have only one centre.

If the sundial is corrected for a wide gnomon follow the two styles down to where they intersect the dial plate and note where the two centres are located. If there is only one centre follow a line at the centre of the gnomon down to where it intersects the dial plate. This is the centre of the sundial. Sometimes this may be tricky if the gnomon widens at the base.

The gnomon might be tapered to a point along the sloped surface. This is the style and will lead to the centre. The gnomon might be a circular rod, although this is more common for a vertical sundial. The style is a line down the centre of the rod and it will lead to the centre of the sundial.

5. The 6 a.m. and 6 p.m. hour lines must be located on a horizontal line and pass through the centre(s) of the sundial. Position a string at the furthest points of these hour lines and pull it tight. The hour lines must not move above or below this string. The string must also pass through the centre(s) of the sundial. If the sundial indicates summer time these hour lines will be numbered 7 a.m. and 7 p.m.

6. The noon (12) hour line(s) must be at right angles or perpendicular to the 6 a.m. and 6 p.m. hour lines. The gnomon must be located on or between the noon line(s). If the sundial indicates summer time the noon line will be numbered 1 p.m.

7. The hour lines for a narrow gnomon originate from a single centre while those for a wide gnomon from two centres. See Figures 7, 8, 9 and 10. Use the plastic ruler to find where the hour lines of the sundial originate from. Check the hour lines for the wide gnomon carefully. The hour lines before 6 a.m. and after 6 p.m. originate from a centre that you might not expect. If you do not compensate for a wide gnomon a significant error can be introduced into the time indicated by a sundial. In reality there is no such thing as a narrow gnomon. Every gnomon requires some thickness in order to give it structure. If a gnomon becomes "thin enough" you may be willing to accept the error. "Thin enough" will depend upon the size of the dial plate and the actual spacing between the hour lines. A gnomon width approaching 1/4 inch (6 mm) for a sundial that is to give an accurate reading is getting to be quite wide.

If you have checked all these points and find the sundial acceptable then the sundial meets the basic design criteria. However, it is not known yet whether the hour lines and gnomon have been designed for a specific latitude and what that latitude is. For the next two checks the horizontal sundial "Check-A-Dial" will be used. With the "Check-A-Dial" you will determine what latitude one of the hour lines is positioned for and the latitude that the gnomon is designed for.

This procedure is illustrated in Figure 11. Note that the sundial design is what you might typically find. It has a single centre and a relatively wide gnomon. The "Check-A-Dial" is shown with "Gnomon Width" band removed.

The sundial in Figure 11 is designed for the Northern Hemisphere. The "Check-A-Dial" being used will indicate what latitude the 3 p.m. hour line is positioned for. Position the "Check-A-Dial" with the arrow marked "Centre" and the centre of the sundial and the "Gnomon Width" edge parallel to the noon (12) hour line. Check where the 3 p.m. hour line intersects the curved edge of the "Check-A-Dial" and record the latitude. If the hour line is between the latitude lines estimate the latitude. If the hour line is too short to reach the "Check-A-Dial" use the plastic ruler to extend it. Now turn the "Check-A-Dial" over and determine the gnomon height. For a horizontal sundial the gnomon height is equal to the latitude that sundial is designed for.

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Horizontal Sundial

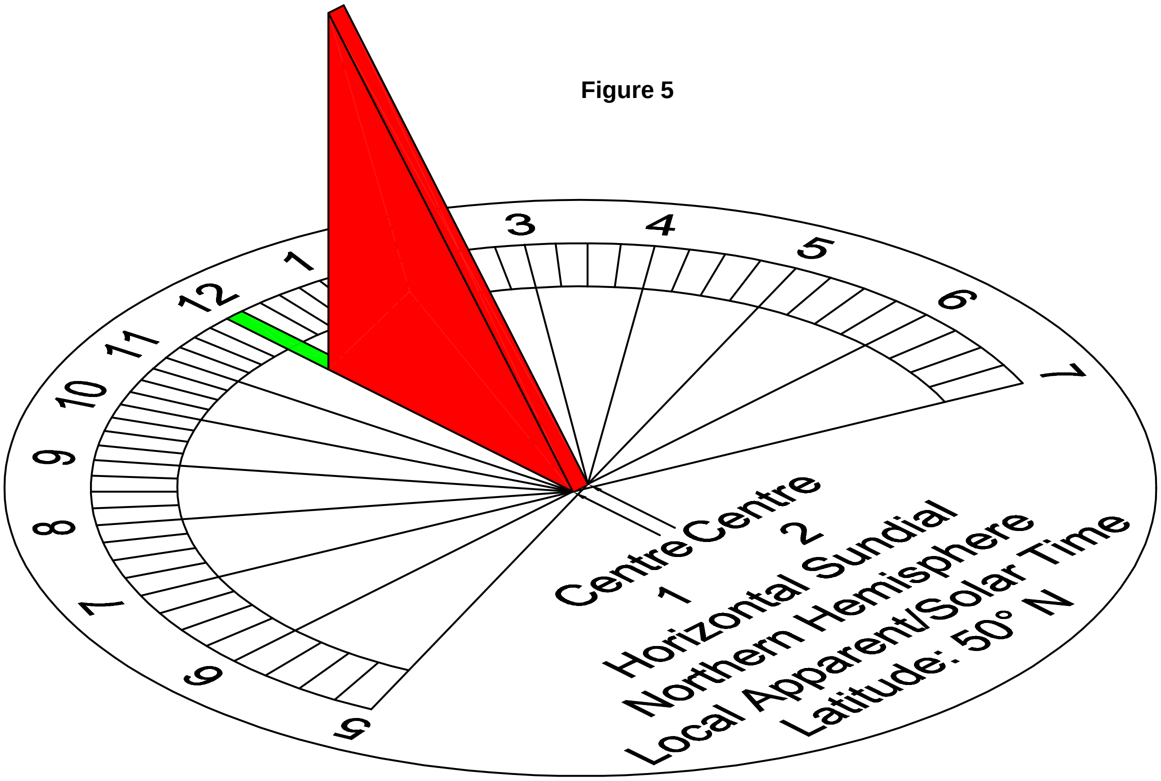

Figure 5

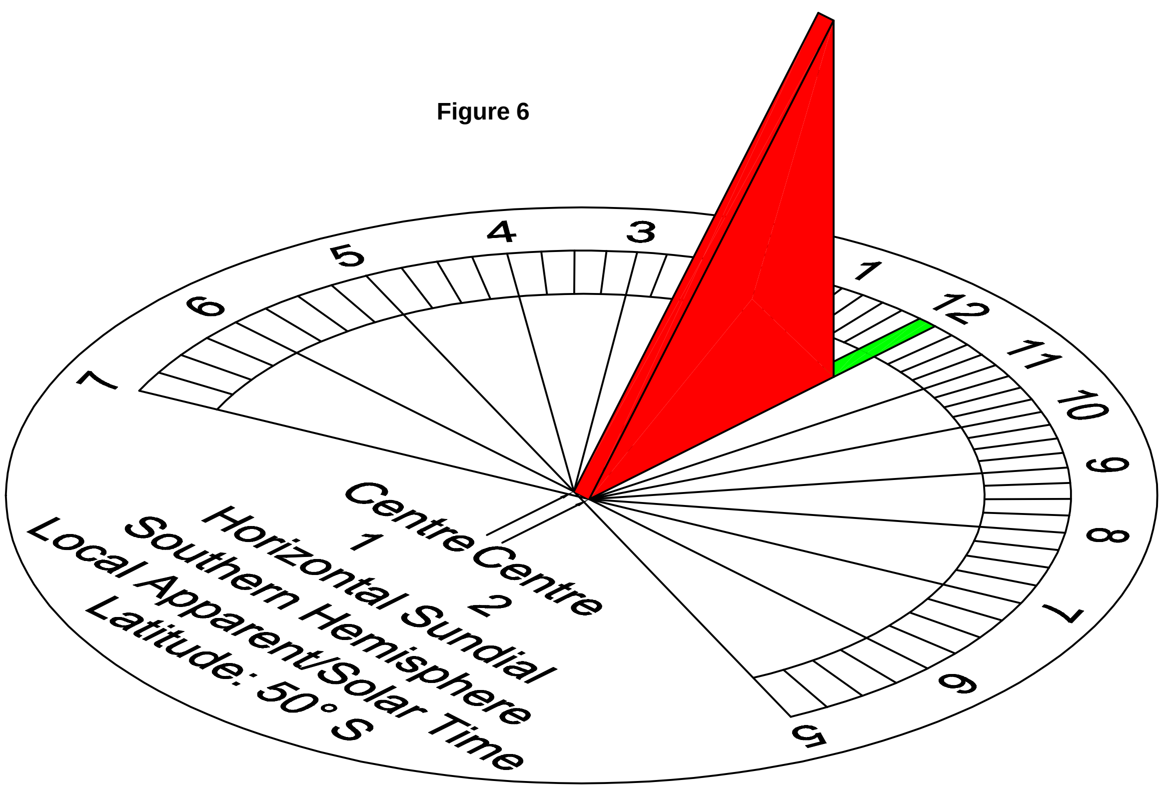

Figure 6

The Sundial Primer - "Check-A-Dial"
Horizontal Sundial

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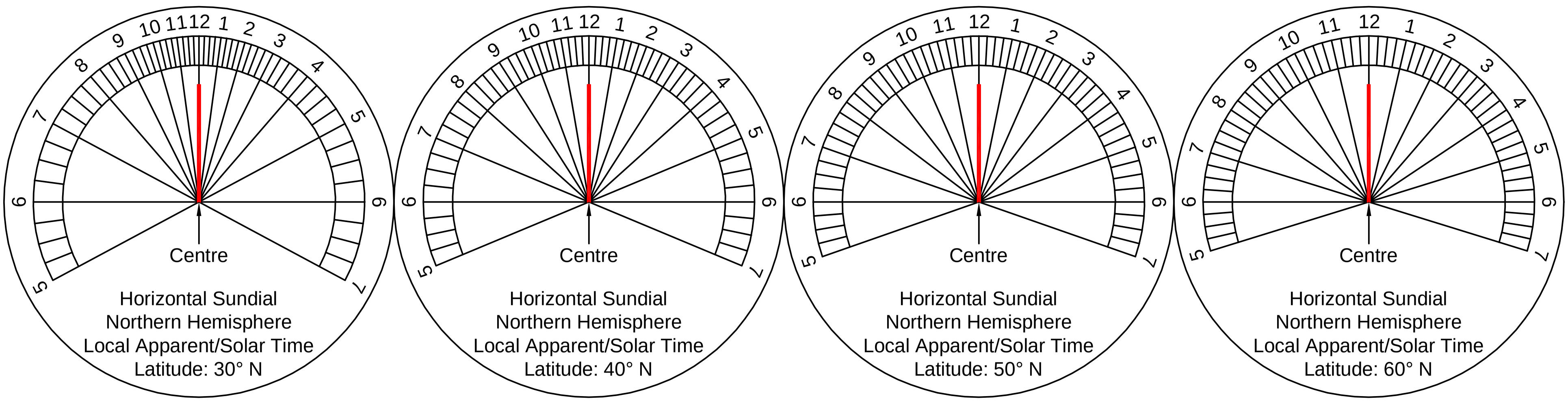
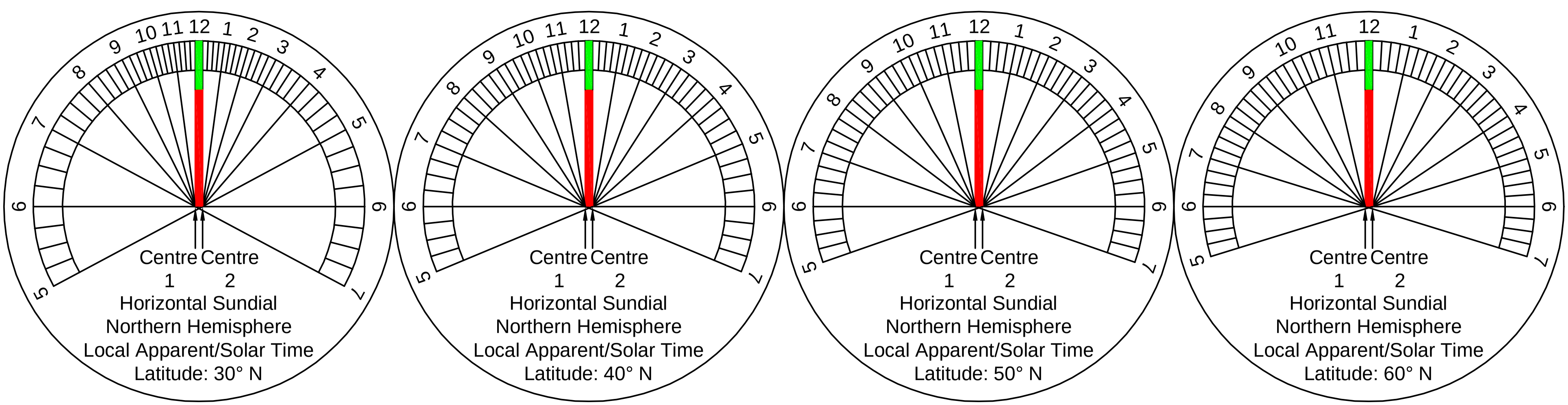


RED - Gnomon
GREEN - Noon Gap

Horizontal Sundial - Northern Hemisphere - Narrow Gnomon
Figure 7



Horizontal Sundial - Northern Hemisphere - Wide Gnomon
Figure 8

The Sundial Primer - "Check-A-Dial"
Horizontal Sundial

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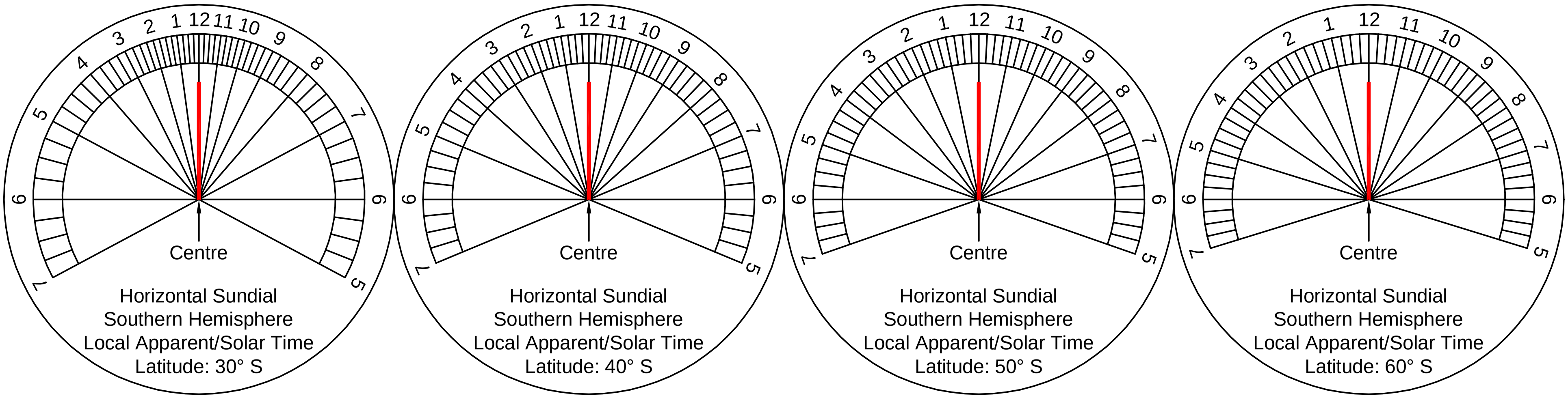
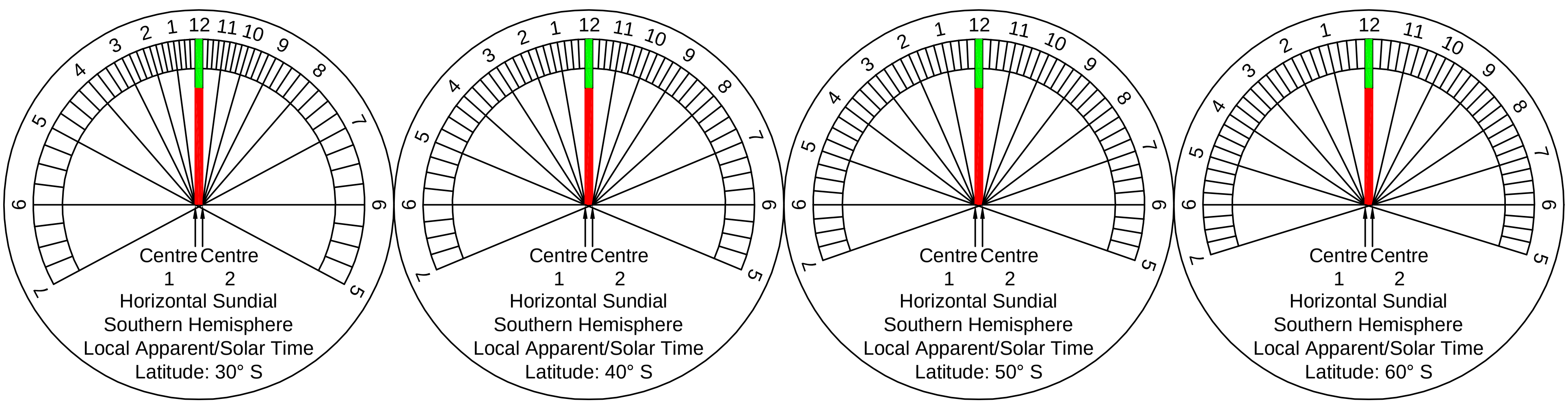


RED - Gnomon
GREEN - Noon Gap

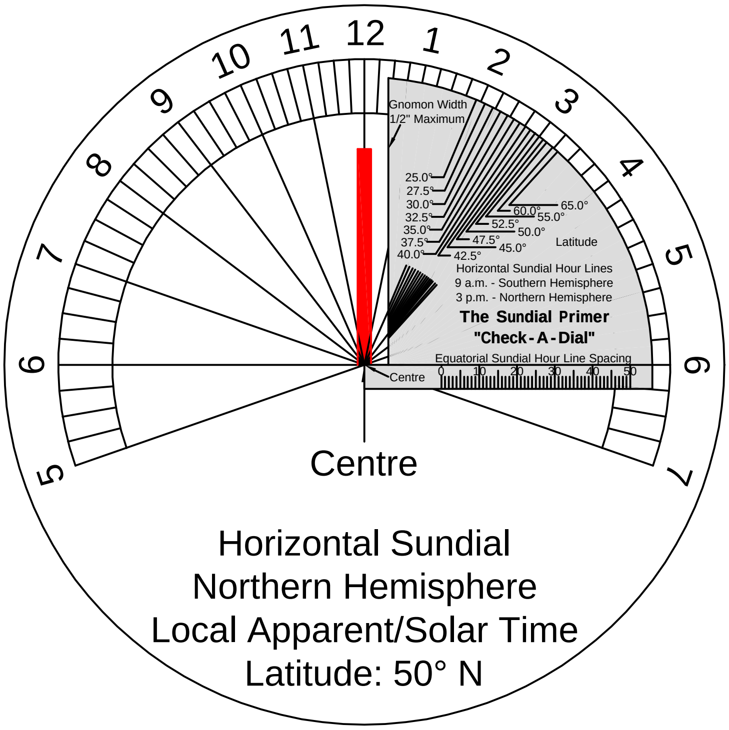
Horizontal Sundial - Southern Hemisphere - Narrow Gnomon
Figure 9



Horizontal Sundial - Southern Hemisphere - Wide Gnomon
Figure 10

The Sundial Primer - "Check-A-Dial" Horizontal Sundial

created by
Carl Sabanski



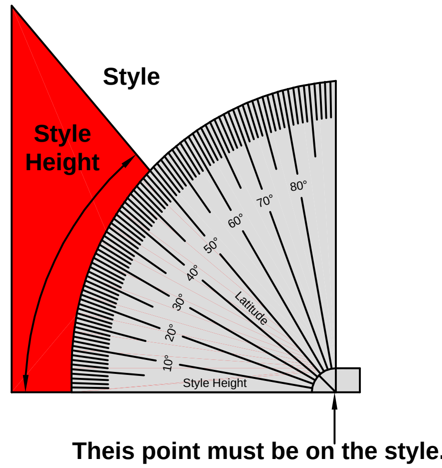


Figure 11

Both latitude values obtained using the "Check-A-Dial" must be equal or very close. If they are then there is a good chance that the sundial has been designed for a specific latitude and you have determined what it is. Unfortunately this is not a guarantee and only the manufacturer knows for sure. Let's say it's good but the latitude does not match yours. This will likely be the case.

A correctly designed horizontal sundial for a given latitude can work at another latitude by tilting the noon (top) edge or the bottom edge. Do not turn the dial plate to make the sundial read correctly. This will only work for the moment and the reading will soon be wrong again. If your latitude is higher than that of the sundial you need to raise the top edge of the dial plate until the gnomon height equals your latitude. This tilt would probably look not too bad as the sundial is being tilted towards you. If your latitude is lower than that of the sundial you need to raise the bottom edge of the dial plate until the gnomon height equals your latitude. This tilt might be acceptable if it is only a few degrees as the sundial is being tilted away from you. This topic will not be discussed further here but check The Sundial Primer "Latitude Correction" page for more information.

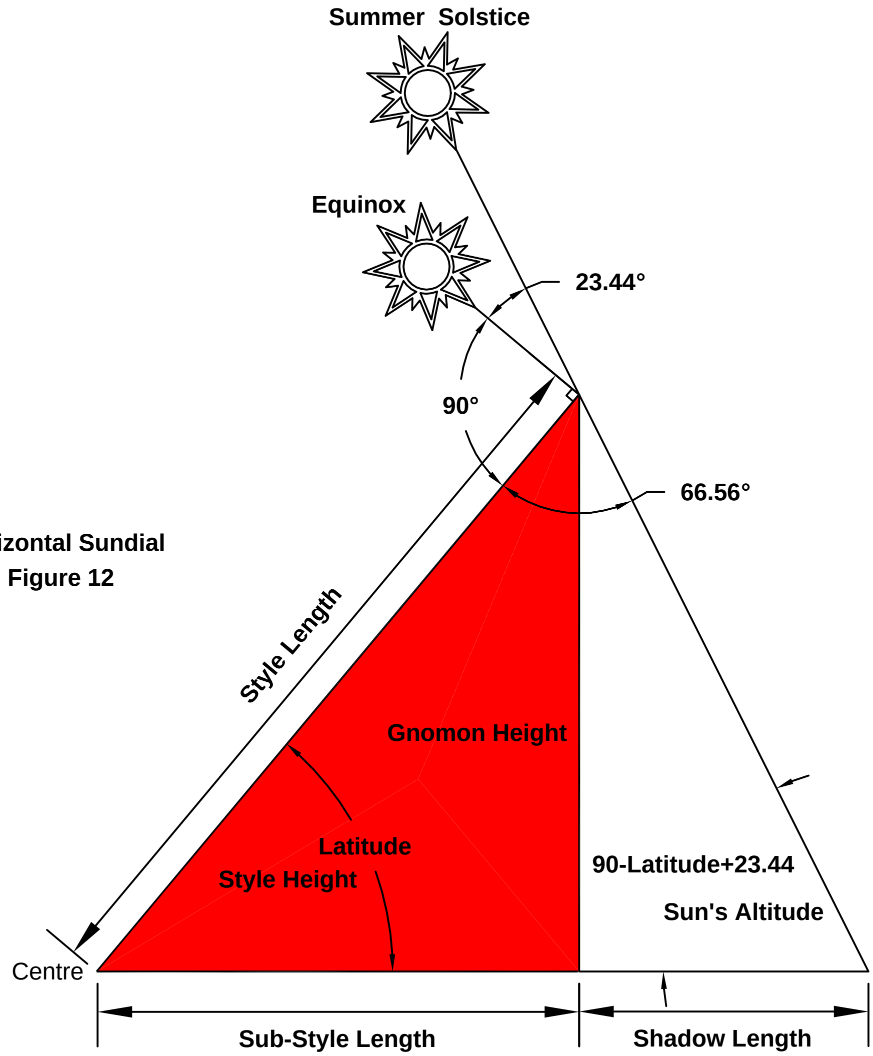
One last item that you should consider is the gnomon height as this will affect the length of shadow available and the ease with which the sundial can be read during certain times of the year.

The gnomon's shadow will be the shortest at solar noon on the summer solstice as illustrated in Figure 12. You can determine this shadow length from Table 1. The value "SMIN" is the minimum shadow length that will be cast for a gnomon height of 1 unit. To determine the shadow length for an actual gnomon measure the gnomon height and multiply by the "SMIN" value for the sundial's latitude. This assumes that the sundial is designed for the specified latitude. As an example let's say we have a horizontal sundial designed for a latitude of 50° and the gnomon height is 4 inches. From the table the "SMIN" value for that latitude is 0.500. The gnomon will cast a shadow at noon on the summer solstice that will be 2 inches (4 x 0.5) long. As the time moves away from noon and the date moves from the summer solstice the shadow will get longer.

The "Style Length" column can be used to determine the length of style required to ensure that it's shadow will always be on an hour line. Measure from the centre of sundial along the noon line to the furthest point the shadow should fall. Obtain the "Style Length" value from Table 1 for the sundial's latitude and multiply it by the measured value. Again let's say we have a horizontal sundial designed for a latitude of 50°. The shadow should extend 9 inches from the centre along the noon line. From the table the "Style Length" value for the latitude is 0.975. The style length required to cast a shadow at noon on the summer solstice that is 9 inches from the centre of the sundial will be 8.8 inches (9 x 0.975) long. The actual style length can be easily measured.

The Sundial Primer - "Check-A-Dial"
Horizontal Sundial

created by
Carl Sabanski



Horizontal Sundial
Figure 12

LATITUDE DEGREES	SMIN	STYLE LENGTH	LATITUDE DEGREES	SMIN	STYLE LENGTH	LATITUDE DEGREES	SMIN	STYLE LENGTH	LATITUDE DEGREES	SMIN	STYLE LENGTH
25	0.027	1.090	35.5	0.214	1.066	45.5	0.405	1.010	55.5	0.626	0.924
25.5	0.036	1.089	36	0.223	1.064	46	0.415	1.007	56	0.639	0.919
26	0.045	1.089	36.5	0.232	1.062	46.5	0.426	1.003	56.5	0.651	0.914
26.5	0.053	1.088	37	0.241	1.060	47	0.436	0.999	57	0.663	0.908
27	0.062	1.088	37.5	0.250	1.057	47.5	0.446	0.995	57.5	0.676	0.903
27.5	0.071	1.087	38	0.260	1.055	48	0.457	0.991	58	0.689	0.898
28	0.080	1.087	38.5	0.269	1.053	48.5	0.468	0.987	58.5	0.702	0.892
28.5	0.089	1.086	39	0.278	1.050	49	0.478	0.983	59	0.715	0.887
29	0.097	1.085	39.5	0.288	1.047	49.5	0.489	0.979	59.5	0.728	0.881
29.5	0.106	1.084	40	0.297	1.045	50	0.500	0.975	60	0.742	0.876
30	0.115	1.083	40.5	0.307	1.042	50.5	0.511	0.971	60.5	0.755	0.870
30.5	0.124	1.082	41	0.316	1.039	51	0.522	0.966	61	0.769	0.864
31	0.133	1.081	41.5	0.326	1.036	51.5	0.533	0.962	61.5	0.783	0.858
31.5	0.142	1.079	42	0.336	1.033	52	0.544	0.957	62	0.797	0.852
32	0.151	1.078	42.5	0.345	1.030	52.5	0.556	0.953	62.5	0.812	0.846
32.5	0.159	1.076	43	0.355	1.027	53	0.567	0.948	63	0.826	0.840
33	0.168	1.075	43.5	0.365	1.024	53.5	0.579	0.943	63.5	0.841	0.834
33.5	0.177	1.073	44	0.375	1.021	54	0.590	0.939	64	0.856	0.828
34	0.186	1.072	44.5	0.385	1.017	54.5	0.602	0.934	64.5	0.871	0.822
34.5	0.195	1.070	45	0.395	1.014	55	0.614	0.929	65	0.887	0.816
35	0.205	1.068									

Table 1

The Sundial Primer - "Check-A-Dial"

created by
Carl Sabanski

Vertical Sundial

The second sundial to be discussed is less common but available. This is the vertical sundial. There are a number of types of vertical sundials but the two that will be dealt with here are the direct south vertical sundial for the Northern Hemisphere and the direct north vertical sundial for the Southern Hemisphere.

Vertical Sundial: any dial in which the dial plate is vertical.

The definitions that apply to the terms illustrated in Figure 13 are the same for both horizontal and vertical sundials. They have been given in the horizontal sundial section and will not be repeated here.

To assist with the following description a number of illustrations have been provided. Figure 14 illustrates a vertical sundial for the Northern Hemisphere and Figure 15 for the Southern Hemisphere. Figures 16, 17, 18 and 19 illustrate a number of sundial layouts for the Northern and Southern Hemispheres. There are vertical sundials shown designed for four different latitudes and with narrow and wide gnomons. Refer to these figures as well as Figure 13 to help you understand what is being discussed.

The following are important details to consider when you are evaluating a solar time vertical sundial that you are considering to purchase.

1. The upper and lower surfaces of the dial plate should flat. Variations in the upper surface, particularly where the hour lines are located, could have an affect on the reading. If there are any raised design features they located below the centre of the sundial they should not be very high. Although they may look nice they could obscure the gnomon's shadow at times. Variations in the lower surface could make it difficult to install the dial plate parallel to the wall.

2. Check that gnomon is securely fastened to the dial plate. The gnomon begins at the centre of the sundial and rises towards the noon (12) marker. All gnomons will have some width and this width must not change along the entire length of the sloped surface where the style(s) is located. The two edges (styles) of the sloped surface must be parallel, clean and sharp in order to cast a good shadow. The width of the gnomon must not increase below the sloped surface other than at the base where it may increase slightly to allow fastening to the dial plate. If wide features are incorporated into the gnomon they will obscure the gnomon's shadow. The gnomon must be straight and not tilted to the left or right.

3. The numbering of the hour lines must be couter clockwise for the Northern Hemisphere and clockwise for the Southern Hemisphere.

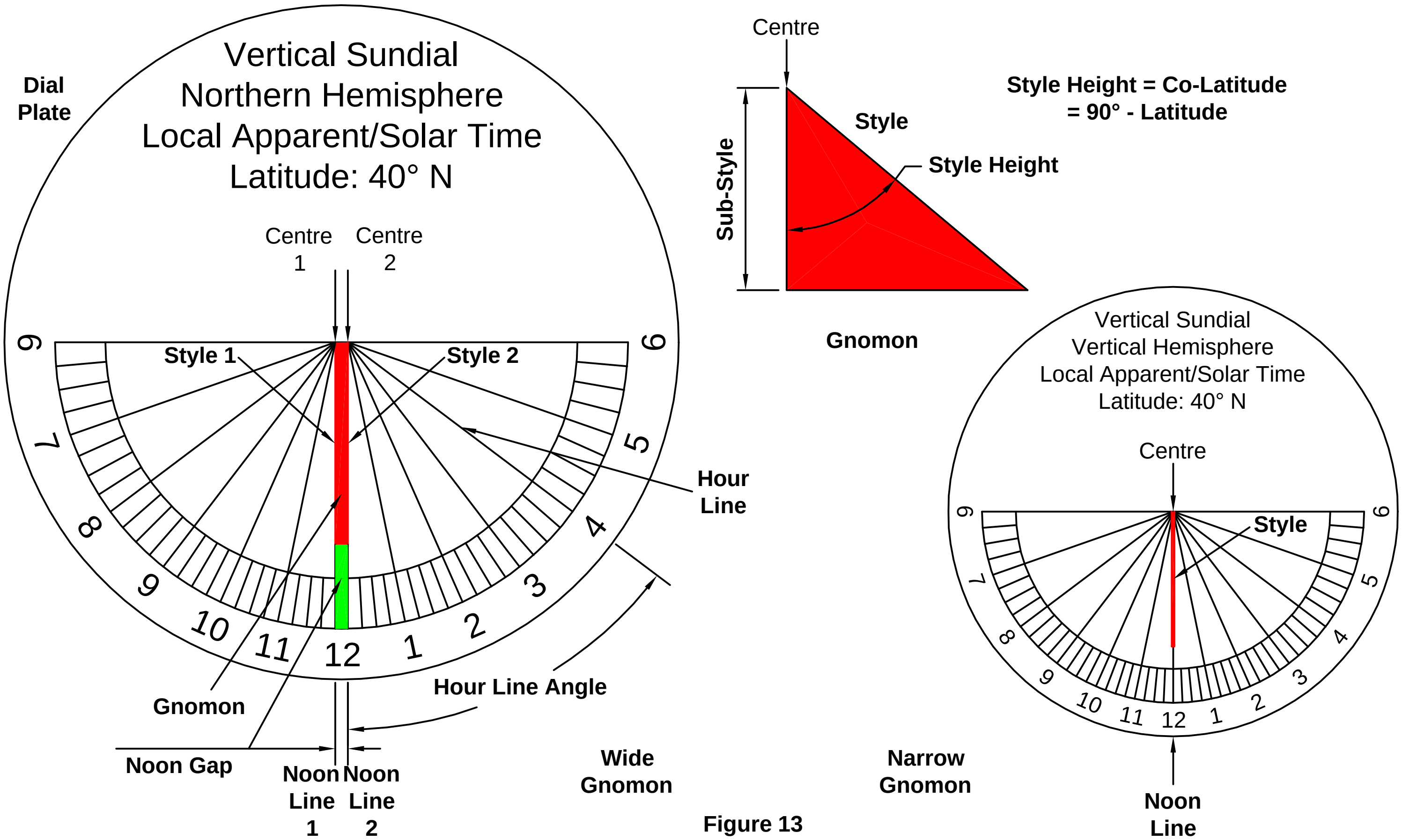


Figure 13

Vertical Sundial

created by
Carl Sabanski

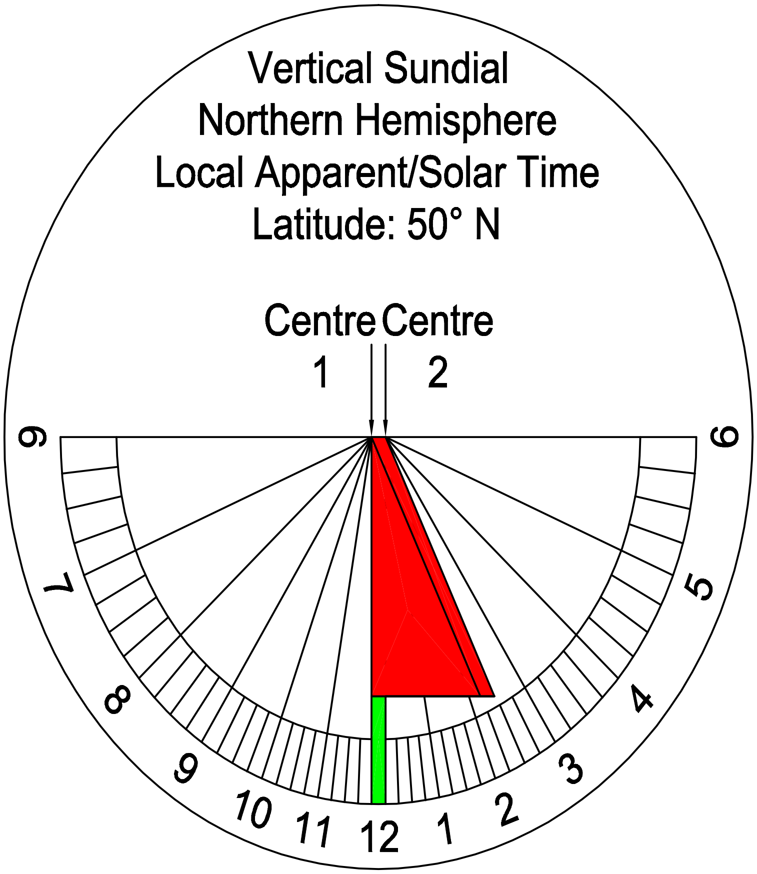


Figure 14

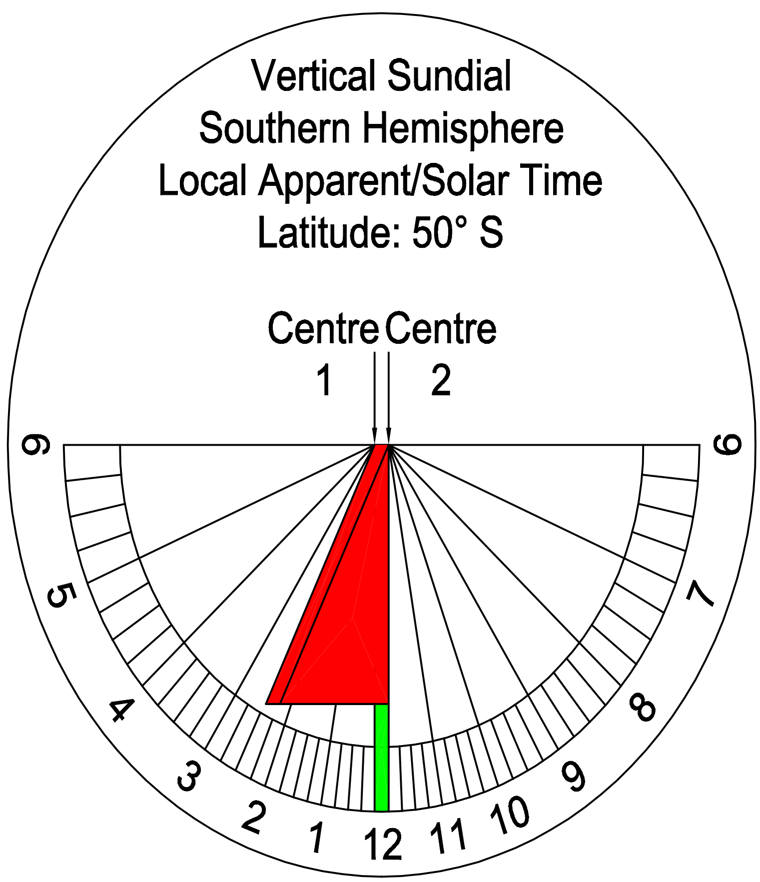
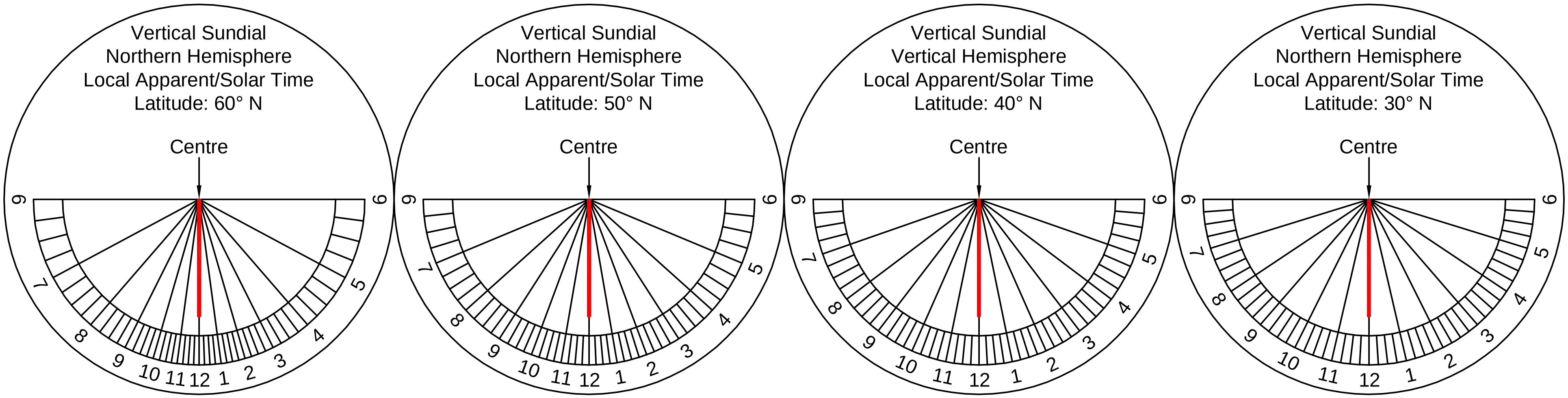


Figure 15

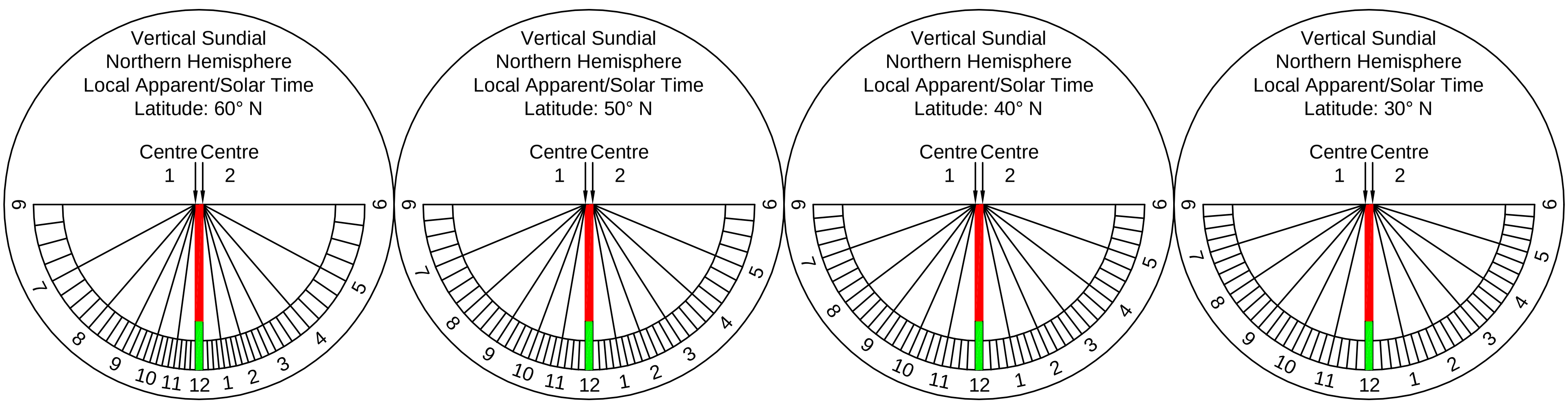
The Sundial Primer - "Check-A-Dial"
Vertical Sundial

created by
Carl Sabanski



RED - Gnomon
GREEN - Noon Gap

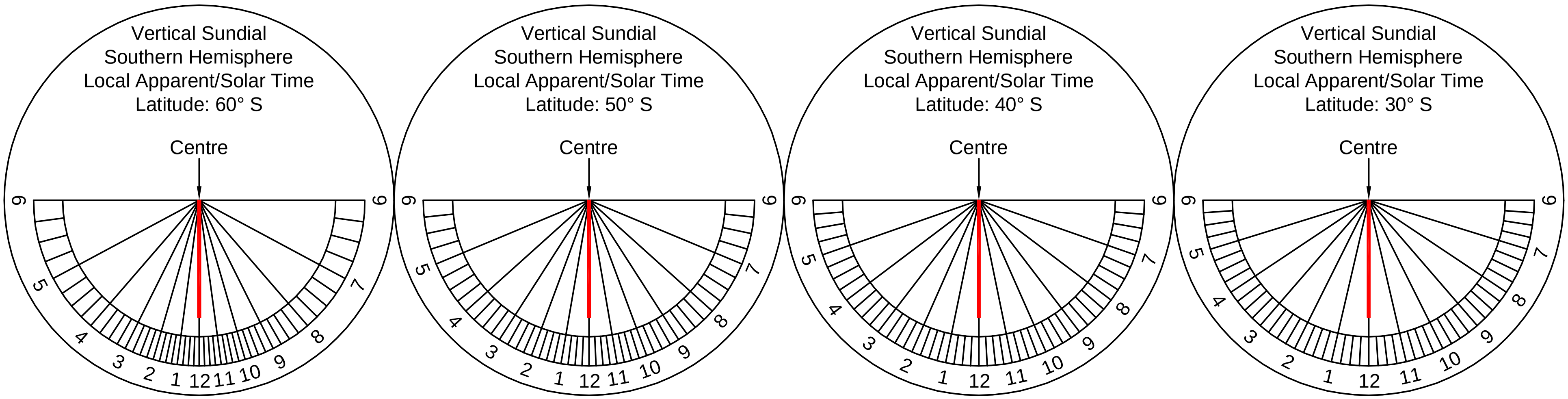
Vertical Sundial - Northern Hemisphere - Narrow Gnomon
Figure 16



Vertical Sundial - Northern Hemisphere - Wide Gnomon
Figure 17

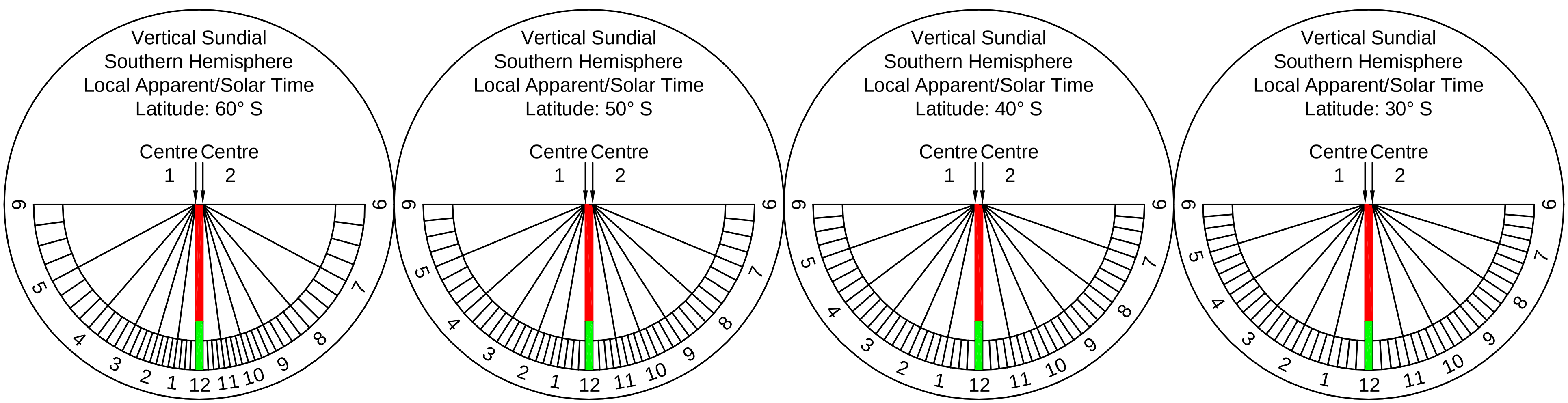
The Sundial Primer - "Check-A-Dial"
Vertical Sundial

created by
Carl Sabanski



RED - Gnomon
GREEN - Noon Gap

Vertical Sundial - Southern Hemisphere - Narrow Gnomon
Figure 18



Vertical Sundial - Southern Hemisphere - Wide Gnomon
Figure 19

4. At this point the centre(s) of the sundial should be found based upon the position of the gnomon. Examine the hour lines near noon to determine whether the sundial design compensates for a thick gnomon. There will be two noon lines with a full hour before and after these lines. See Figures 17 and 19. Check this carefully. In most cases the sundial is likely to have only one centre.

If the sundial is corrected for a wide gnomon follow the two styles down to where they intersect the dial plate and note where the two centres are located. If there is only one centre follow a line at the centre of the gnomon down to where it intersects the dial plate. This is the centre of the sundial. Sometimes this may be tricky if the gnomon widens at the base.

The gnomon might be tapered to a point along the sloped surface. This is the style and will lead to the centre. The gnomon might be a circular rod, which is common for a vertical sundial. The style is a line down the centre of the rod and it will lead to the centre of the sundial.

5. The 6 a.m. and 6 p.m. hour lines must be located on a horizontal line and pass through the centre(s) of the sundial. Position a string at the furthest points of these hour lines and pull it tight. The hour lines must not move above or below this string. The string must also pass through the centre(s) of the sundial. If the sundial indicates summer time these hour lines will be numbered 7 a.m. and 7 p.m.

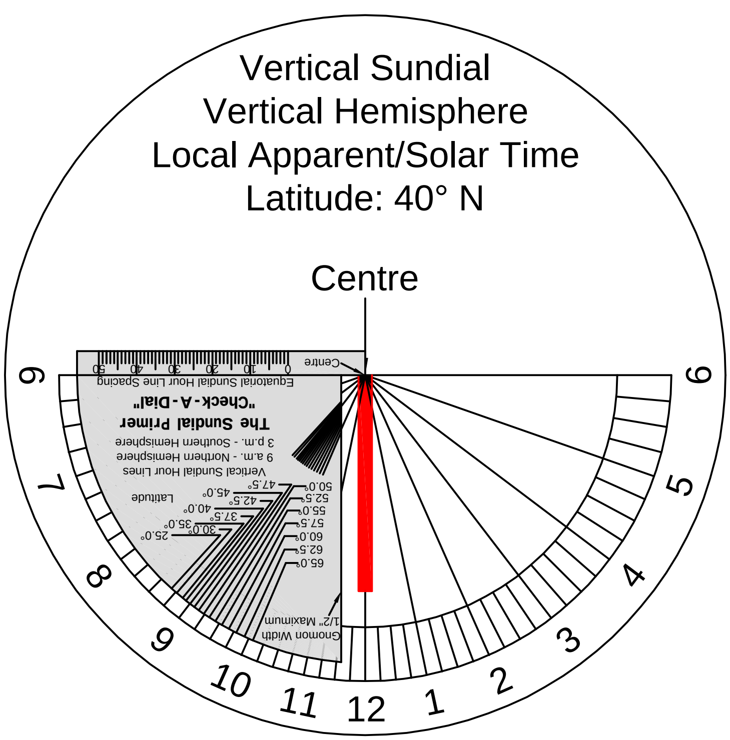
6. The noon (12) hour line(s) must be at right angles or perpendicular to the 6 a.m. and 6 p.m. hour lines. The gnomon must be located on or between the noon line(s). If the sundial indicates summer time the noon line will be numbered 1 p.m.

7. The hour lines for a narrow gnomon originate from a single centre while those for a wide gnomon from two centres. See Figures 16, 17, 18 and 19. Use the plastic ruler to find where the hour lines of the sundial originate from. If you do not compensate for a wide gnomon a significant error can be introduced into the time indicated by a sundial. In reality there is no such thing as a narrow gnomon. Every gnomon requires some thickness in order to give it structure. If a gnomon becomes "thin enough" you may be willing to accept the error. "Thin enough" will depend upon the size of the dial plate and the actual spacing between the hour lines. A gnomon width approaching 1/4 inch (6 mm) for a sundial that is to give an accurate reading is getting to be quite wide.

8. A direct south vertical sundial in the Northern Hemisphere and direct north vertical sundial in the Southern Hemisphere will indicate the time for a maximum of 12 hours from 6 a.m. to 6 p.m. This will occur on two days only, the equinoxes. The gnomon's shadow will never fall on the hour lines before 6 a.m. and after 6 p.m. so they serve no purpose.

If you have checked all these points and find the sundial acceptable then the sundial meets the basic design criteria. However, it is not known yet whether the hour lines and gnomon have been designed for a specific latitude and what that latitude is. For the next two checks the vertical sundial "Check-A-Dial" will be used. With the "Check-A-Dial" you will determine what latitude one of the hour lines is positioned for and the style height that the gnomon is designed for.

This procedure is illustrated in Figure 20. Note that the sundial design is what you might typically find. It has a single centre and a relatively wide gnomon. The "Check-A-Dial" is shown with "Gnomon Width" band removed.



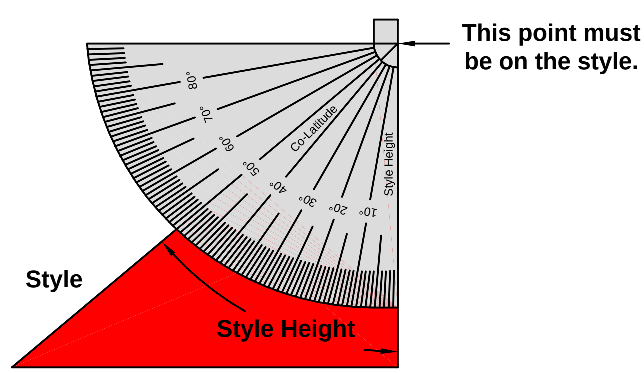


Figure 20

The Sundial Primer - "Check-A-Dial" Vertical Sundial

created by
Carl Sabanski

The sundial in Figure 20 is designed for the Northern Hemisphere. The "Check-A-Dial" being used will indicate what latitude the 9 a.m. hour line is positioned for. Position the "Check-A-Dial" with the arrow marked "Centre" and the centre of the sundial and the "Gnomon Width" edge parallel to the noon (12) hour line. Check where the 3 p.m. hour line intersects the curved edge of the "Check-A-Dial" and record the latitude. If the hour line is between the latitude lines estimate the latitude. If the hour line is too short to reach the "Check-A-Dial" use the plastic ruler to extend it. Now turn the "Check-A-Dial" over and determine the gnomon height. For a vertical sundial the gnomon height does not equal the latitude that sundial is designed for and so these two numbers are different. The gnomon height is equal to the co-latitude or 90° minus the latitude.

The latitude values obtained using the "Check-A-Dial" is used to calculate the co-latitude and this value must be equal or very close to the gnomon height obtained using the "Check-A-Dial". If they are then there is a good chance that the sundial has been designed for a specific latitude and you have determined what it is. Unfortunately this is not a guarantee and only the manufacturer knows for sure. Let's say it's good but the latitude does not match yours. This will likely be the case.

A correctly designed vertical sundial for a given latitude can work at another latitude by tilting the noon (bottom) edge or the top edge. Do not turn one side of the dial plate away from the wall to make the sundial read correctly. This will only work for the moment and the reading will soon be wrong again. If your latitude is higher than that of the sundial, resulting in a lower co-latitude, you need to raise the top edge of the dial plate until the gnomon height equals your latitude. This tilt would probably look not too bad as the sundial is being tilted towards you. If your latitude is lower than that of the sundial, resulting in a higher co-latitude, you need to raise the bottom edge of the dial plate until the gnomon height equals your latitude. This tilt might be acceptable if it is only a few degrees as the sundial is being tilted away from you. This topic will not be discussed further here.

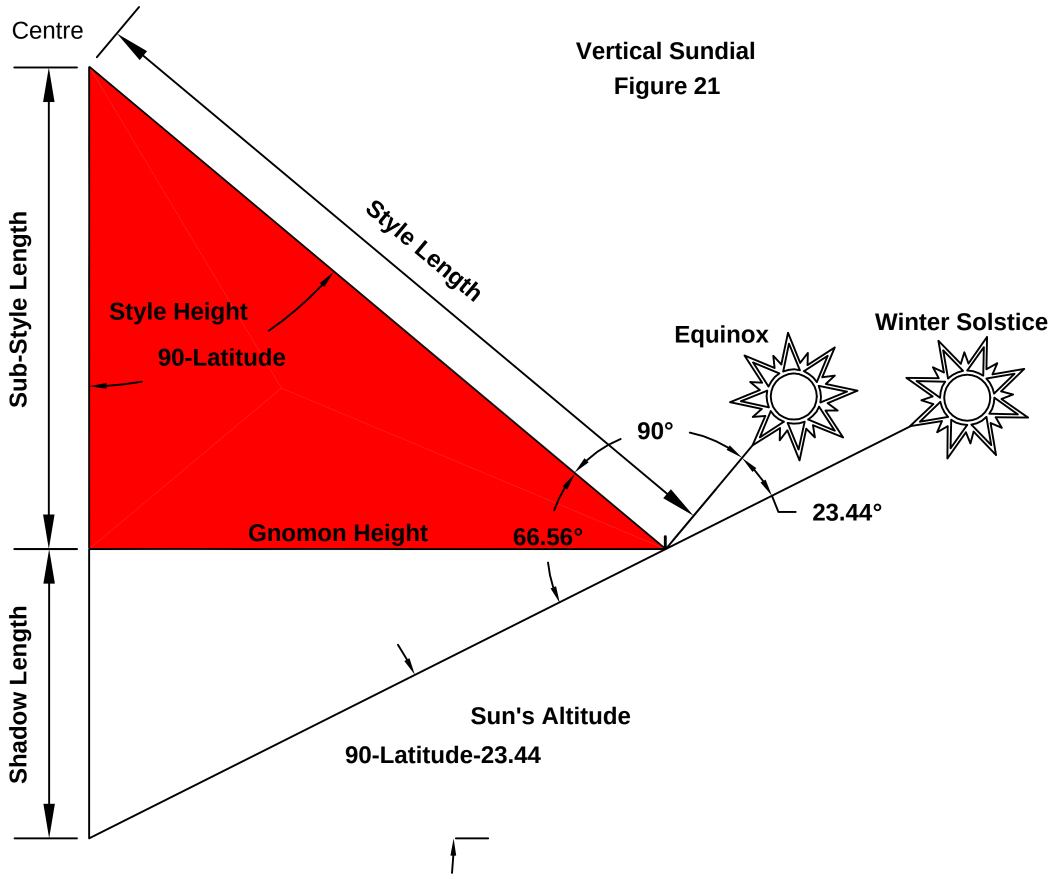
One last item that you should consider is the gnomon height as this will affect the length of shadow available and the ease with which the sundial can be read during certain times of the year.

The gnomon's shadow will be the shortest at solar noon on the winter solstice as illustrated in Figure 21. You can determine this shadow length from Table 2. The value "SMIN" is the minimum shadow length that will be cast for a gnomon height of 1 unit. To determine the shadow length for an actual gnomon measure the gnomon height and multiply by the "SMIN" value for the sundial's latitude. This assumes that the sundial is designed for the specified latitude. As an example let's say we have a vertical sundial designed for a latitude of 40° and the gnomon height is 4 inches. From the table the "SMIN" value for that latitude is 0.500. The gnomon will cast a shadow at noon on the summer solstice that will be 2 inches (4 x 0.5) long. As the time moves away from noon and the date moves from the winter solstice the shadow will get longer.

The "Style Length" column can be used to determine the length of style required to ensure that it's shadow will always be on an hour line. Measure from the centre of sundial along the noon line to the furthest point the shadow should fall. Obtain the "Style Length" value from Table 2 for the sundial's latitude and multiply it by the measured value. Again let's say we have a vertical sundial designed for a latitude of 40°. The shadow should extend 9 inches from the centre along the noon line. From the table the "Style Length" value for the latitude is 0.975. The style length required to cast a shadow at noon on the winter solstice that is 9 inches from the centre of the sundial will be 8.8 inches (9 x 0.975) long. The actual style length can be easily measured.

The Sundial Primer - "Check-A-Dial"
Vertical Sundial

created by
Carl Sabanski



LATITUDE DEGREES	SMIN	STYLE LENGTH	LATITUDE DEGREES	SMIN	STYLE LENGTH	LATITUDE DEGREES	SMIN	STYLE LENGTH	LATITUDE DEGREES	SMIN	STYLE LENGTH
25	0.887	0.816	35.5	0.602	0.934	45.5	0.385	1.017	55.5	0.195	1.070
25.5	0.871	0.822	36	0.590	0.939	46	0.375	1.021	56	0.186	1.072
26	0.856	0.828	36.5	0.579	0.943	46.5	0.365	1.024	56.5	0.177	1.073
26.5	0.841	0.834	37	0.567	0.948	47	0.355	1.027	57	0.168	1.075
27	0.826	0.840	37.5	0.556	0.953	47.5	0.345	1.030	57.5	0.159	1.076
27.5	0.812	0.846	38	0.544	0.957	48	0.336	1.033	58	0.151	1.078
28	0.797	0.852	38.5	0.533	0.962	48.5	0.326	1.036	58.5	0.142	1.079
28.5	0.783	0.858	39	0.522	0.966	49	0.316	1.039	59	0.133	1.081
29	0.769	0.864	39.5	0.511	0.971	49.5	0.307	1.042	59.5	0.124	1.082
29.5	0.755	0.870	40	0.500	0.975	50	0.297	1.045	60	0.115	1.083
30	0.742	0.876	40.5	0.489	0.979	50.5	0.288	1.047	60.5	0.106	1.084
30.5	0.728	0.881	41	0.478	0.983	51	0.278	1.050	61	0.097	1.085
31	0.715	0.887	41.5	0.468	0.987	51.5	0.269	1.053	61.5	0.089	1.086
31.5	0.702	0.892	42	0.457	0.991	52	0.260	1.055	62	0.080	1.087
32	0.689	0.898	42.5	0.446	0.995	52.5	0.250	1.057	62.5	0.071	1.087
32.5	0.676	0.903	43	0.436	0.999	53	0.241	1.060	63	0.062	1.088
33	0.663	0.908	43.5	0.426	1.003	53.5	0.232	1.062	63.5	0.053	1.088
33.5	0.651	0.914	44	0.415	1.007	54	0.223	1.064	64	0.045	1.089
34	0.639	0.919	44.5	0.405	1.010	54.5	0.214	1.066	64.5	0.036	1.089
34.5	0.626	0.924	45	0.395	1.014	55	0.205	1.068	65	0.027	1.090
35	0.614	0.929									

Table 2

The Sundial Primer - "Check-A-Dial" Equatorial Sundial

created by
Carl Sabanski

The final sundial to be discussed is the equatorial sundial. Two specific versions of this sundial, the equatorial ring and armillary sundials, will be discussed. These sundials function in the same manner but the armillary sundial is a more elaborate version of the equatorial ring sundial.

Equatorial Sundial: a dial in which the dial plate is parallel to the equatorial plane and the polar-pointing gnomon is perpendicular to it. The dial has hour lines spaced at 15° intervals. Sunlight falls on the underside of the dial plate from the autumnal equinox until the vernal equinox. For this period, the gnomon must project below the dial plate, which is delineated on both sides. An alternative form replaces the dial plate by a narrow hour ring (or half-ring) allowing the scale, inscribed on its inner circumference, to be read throughout the year.

It is the alternative form or equatorial ring sundial that will be examined here. The gnomon for this sundial is a rod.

Armillary Sundial (or Armillary Sphere): a form of equinoctial (equatorial) sundial which comprises, as a minimum, two circular bands plus a rod through the poles representing the Earth's axis and acting as the gnomon. One band represents the equator (carrying the hour scale) and the other the local meridian. Usually, other great circles are added representing there Prime meridian and the ecliptic plane, sometimes together with small circles for the tropics and arctic circles. These add artistically, but detract from its clarity as a dial.

As both these sundials tell the time in a similar fashion, the criteria developed will apply to both.

To assist with the following description a number of illustrations have been provided. Figure 22 illustrates the layouts of equatorial ring sundials for the Northern Hemisphere the Southern Hemisphere. Figure 23 illustrates an equatorial ring sundial for the Northern Hemisphere. Although it may not look so, the two rings are actually circular. The sundial for the Southern Hemisphere would be identical except the hour lines would be numbered in reverse. Refer to these figures to help you understand what is being discussed.

The following are important details to consider when you are evaluating a solar time equatorial ring or armillary sundial that you are considering to purchase.

1. The hour ring must be circular. If it is not the layout of the hour lines will be affected and also difficult to confirm.

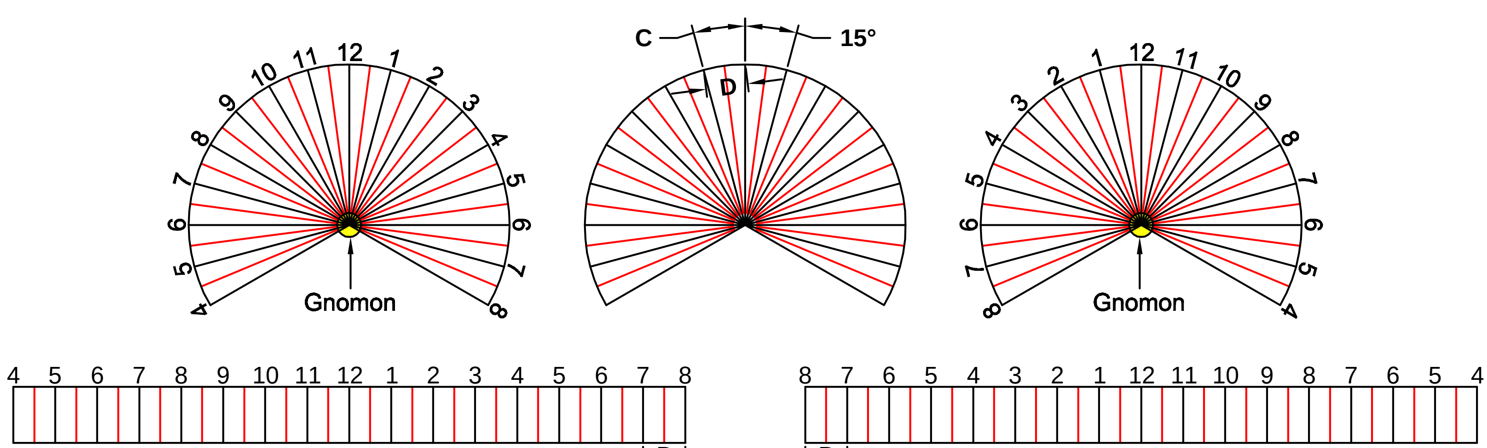
2. The gnomon, which is normally a circular rod, must pass through the centre of the hour ring and be perpendicular to the plane of the hour ring. Check Figure 23.

3. The 6 a.m. and 6 p.m. hour lines must be located on a line that passes through the centres of the hour ring and gnomon. Position a string at the centre of these hour lines and pull it tight. There may be no hour lines as they may be replace by the number 6. The string must want to pass through the centre of the gnomon. If the sundial indicates summer time these hour lines will be numbered 7 a.m. and 7 p.m.

4. The noon (12) hour line(s) must be at right angles or perpendicular to the 6 a.m. and 6 p.m. hour lines. If the sundial indicates summer time the noon line will be numbered 1 p.m.

5. There must only be the hours from 6 a.m. to 6 p.m. located on half (180°) of the hour ring. Any hour lines before 6 a.m. and after 6 p.m. must be located on the other half of the hour ring. This can be clearly seen in Figure 22. If the hour ring is extended to include additional hours, during periods near the equinoxes the ring will obstruct the sun for some time in the morning and the evening.

6. The circumferential distance "C" and the linear distance "D" between any equal time intervals (15 min., 30 min., 1 hour) must be equal. he linear scale provided on all versions of the "Check-A-Dial" can be used to confirm this.

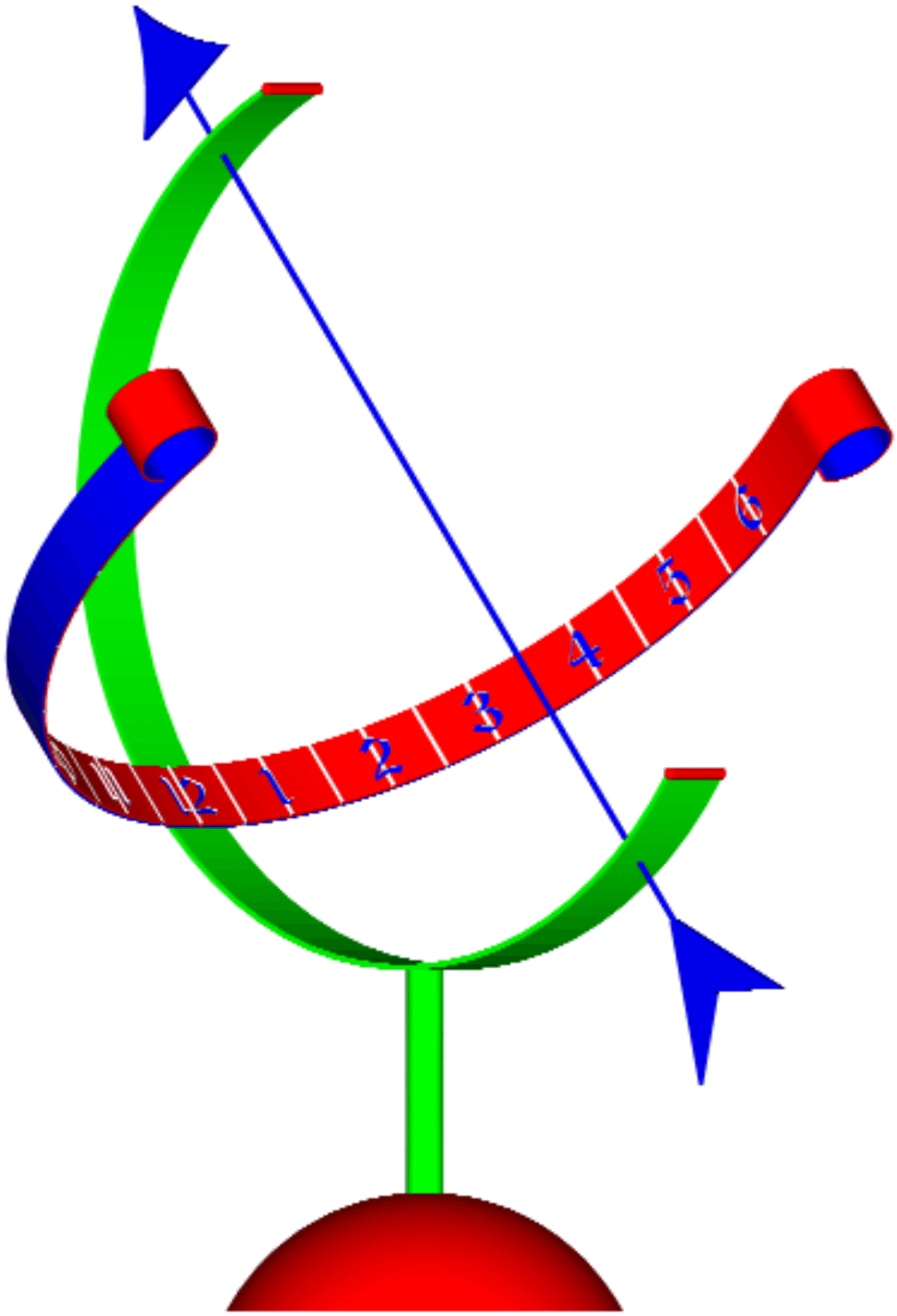


Equatorial Ring Sundial - Northern Hemisphere

Equatorial Ring Sundial - Southern Hemisphere

Figure 22

created by
Carl Sabanski



Equatorial Ring Sundial – Northern Hemisphere
Figure 23

The Sundial Primer - "Check-A-Dial" Equatorial Sundial

created by
Carl Sabanski

The equatorial ring and armillary sundials are universal sundials. They will function at any latitude with only one adjustment. The gnomon must be adjusted to an angle equal to the latitude. This is shown in Figure 24. These sundials are available with bases that allow the sundial to be pivoted. The gnomon can then be easily adjusted.

If the sundial is permanently fixed to the base it will be set to a particular latitude. This can be measured using a protractor. The sundial can be tilted to the local latitude. This will not look good so find a sundial that has a pivot. You will be glad you did.

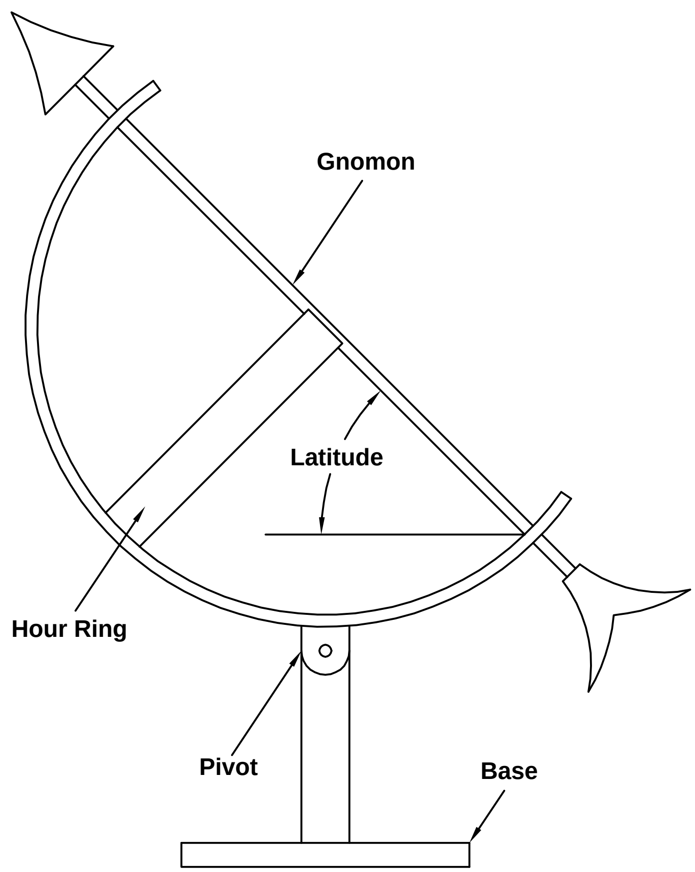


Figure 24

Equatorial ring sundials that include hours before 6 a.m and after 6 p.m. can result in a slight problem. An attempt to illustrate this is made in Figure 25. In this figure the sundial from Figure 24 is viewed with the hour ring positioned on a horizontal plane. This being an equatorial sundial the plane of the hour ring is parallel to the equatorial plane. The gnomon is perpendicular to the hour ring, passes through the centre of the sundial and points to the celestial pole. The meridian ring and the gnomon lie in the plane of the local meridian. The sun is parallel to the hour ring at the equinoxes and moves 23.44° above the hour ring at the summer solstice and 23.44° below the hour ring at the winter solstice.

Equatorial Sundial

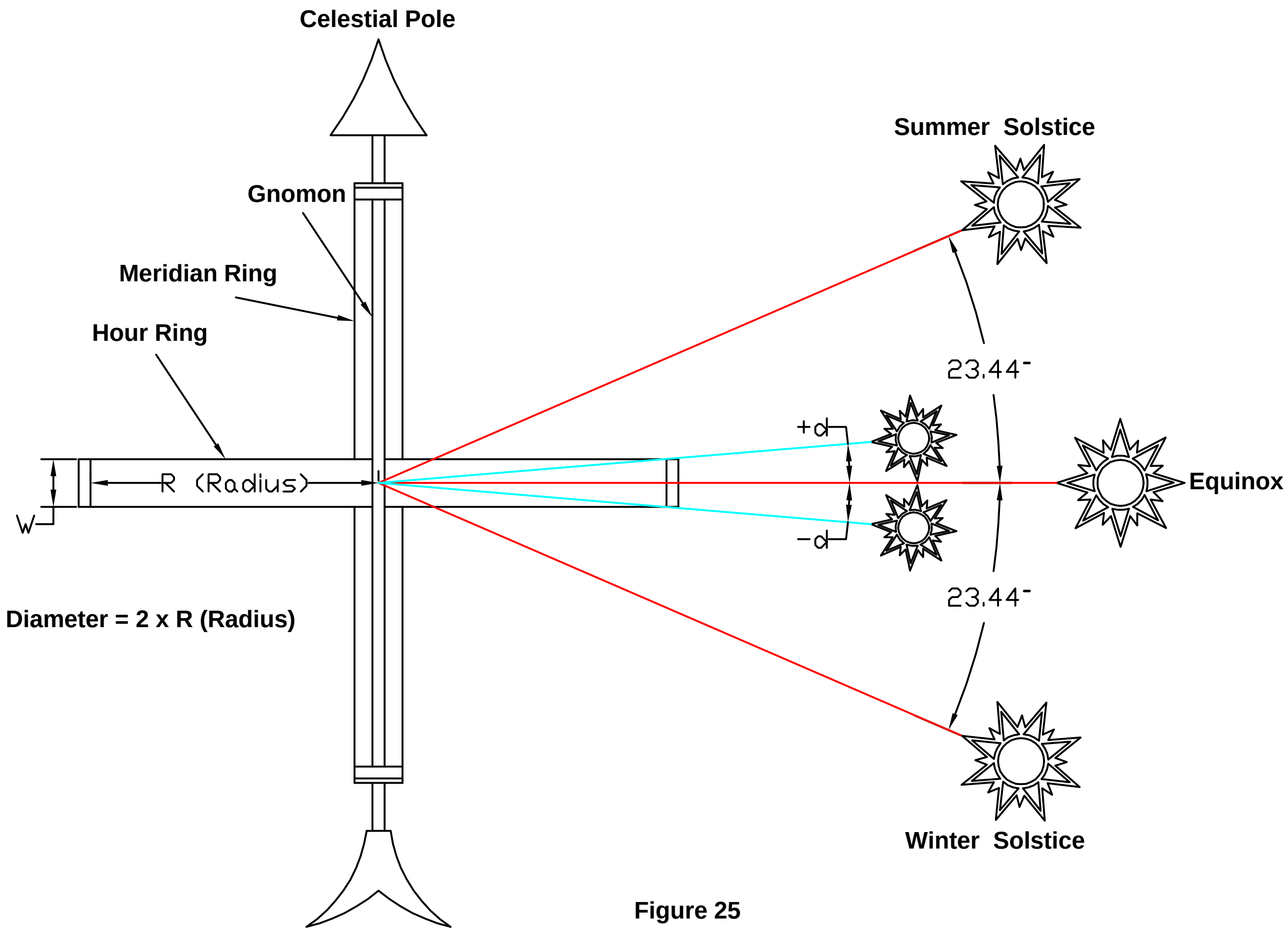


Figure 25

As can be seen from the figure the sun's declination must be "±d" in order that it be above or below the hour ring. In the case of an equatorial ring sundial with an hour range from 6 a.m. to 6 p.m. this is not an issue and the sundial will indicate the time no matter what declination the sun is at. However when the equatorial ring is extended and hours before 6 a.m. and after 6 p.m. are included the hour ring will obscure the sun during certain times of the year. This period is determined by the declination value "d". The sun is not obscured for the entire day unless the hour ring is a complete circle as would be found on an armillary sphere. The time period is determined by the number of additional hours before 6 a.m. and after 6 p.m. For example, if 2 additional hours are added before 6 a.m. and after 6 p.m. the sundial will not be able to indicate these two hours as well as an equivalent number of hours after 6 a.m. and before 6 p.m. The hours from 4 a.m. to 8 a.m. and 4 p.m. to 8 p.m. would not be available for certain periods of the year.

The periods of unavailability are affected by the diameter "D" and the width "W" of the hour ring. Figure 26 and Table 3 are provided to help determine these periods. The graph in Figure 26 is used to determine the sun's declination "d" required to bring it above or below the hour ring. Calculate the ratio of the sundial's hour ring diameter "D" to the width "W" (D / W) of the hour ring. From the graph determine the required sun's declination "d'.

Table 3 is the sun's declination for each day of the year. Find the days when the sun's declination is between the values "±d". There will be two periods when this will occur, around the spring and fall equinoxes.

Let's look at the equatorial ring sundial with hour lines as shown in Figure 22. Say the hour ring has a diameter of 8 inches and a width of 1 inch. The ratio "D / W" is equal to 8. From the graph in Figure 26 the required sun's declination "d" is slightly over 7°. Now go to Table 3 and find the days when the sun's declination is between ±7°. The two periods are March 2 to April 8 and September 4 to October 11. For about 2 months of the year the sundial will not be able to indicate the times from 4 a.m. to 8 a.m. and 4 p.m. to 8 p.m. During the remainder of the year all the hour lines will be functional.

If the hour ring was a full 360° the sundial would function during this period.

The Sundial Primer - "Check-A-Dial" Equatorial Sundial

created by
Carl Sabanski

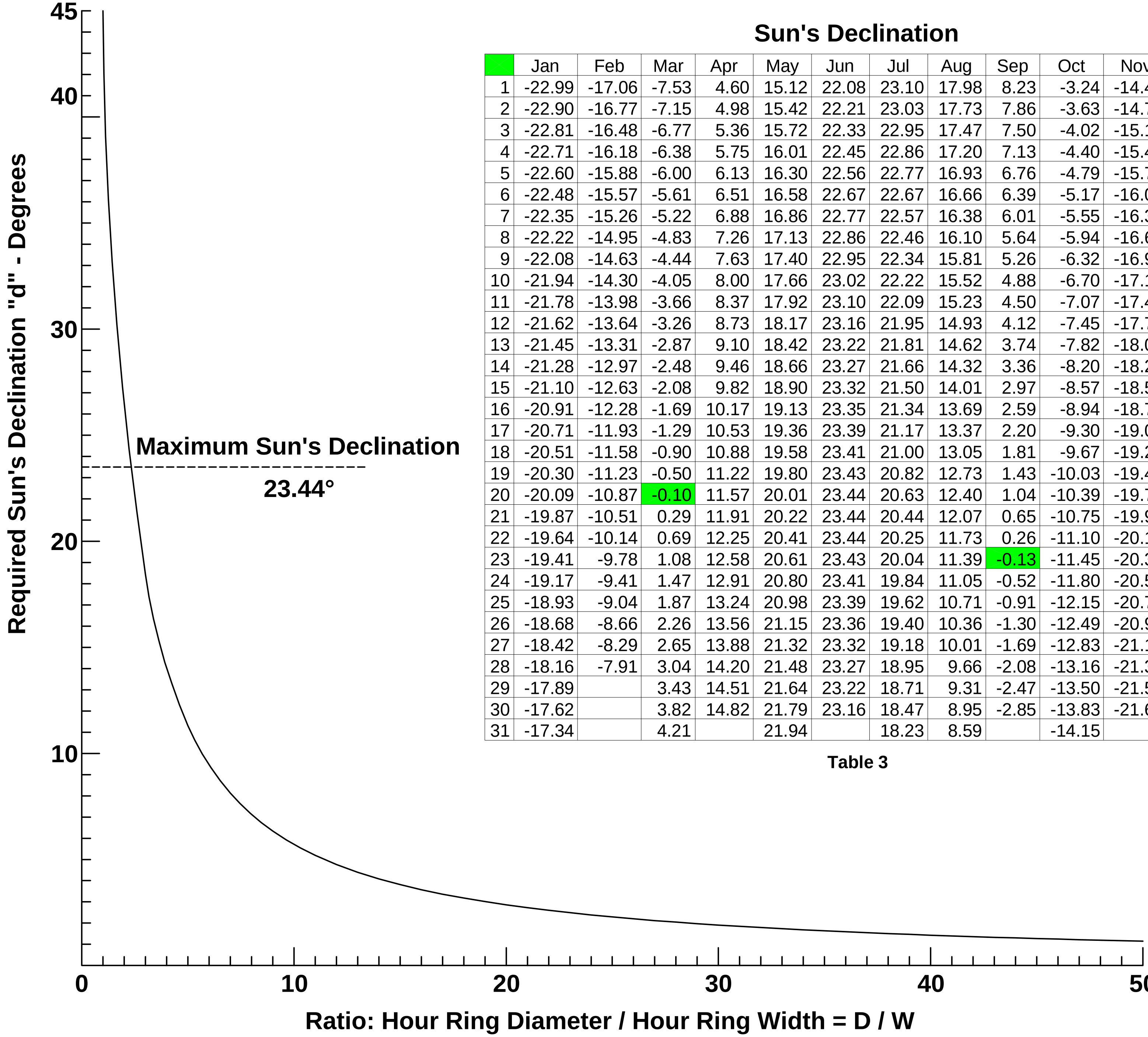


Figure 26

Sun's Declination

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1	-22.99	-17.06	-7.53	4.60	15.12	22.08	23.10	17.98	8.23	-3.24	-14.47	-21.82
2	-22.90	-16.77	-7.15	4.98	15.42	22.21	23.03	17.73	7.86	-3.63	-14.79	-21.97
3	-22.81	-16.48	-6.77	5.36	15.72	22.33	22.95	17.47	7.50	-4.02	-15.11	-22.12
4	-22.71	-16.18	-6.38	5.75	16.01	22.45	22.86	17.20	7.13	-4.40	-15.42	-22.26
5	-22.60	-15.88	-6.00	6.13	16.30	22.56	22.77	16.93	6.76	-4.79	-15.72	-22.39
6	-22.48	-15.57	-5.61	6.51	16.58	22.67	22.67	16.66	6.39	-5.17	-16.02	-22.51
7	-22.35	-15.26	-5.22	6.88	16.86	22.77	22.57	16.38	6.01	-5.55	-16.32	-22.62
8	-22.22	-14.95	-4.83	7.26	17.13	22.86	22.46	16.10	5.64	-5.94	-16.61	-22.73
9	-22.08	-14.63	-4.44	7.63	17.40	22.95	22.34	15.81	5.26	-6.32	-16.90	-22.83
10	-21.94	-14.30	-4.05	8.00	17.66	23.02	22.22	15.52	4.88	-6.70	-17.18	-22.92
11	-21.78	-13.98	-3.66	8.37	17.92	23.10	22.09	15.23	4.50	-7.07	-17.46	-23.01
12	-21.62	-13.64	-3.26	8.73	18.17	23.16	21.95	14.93	4.12	-7.45	-17.73	-23.09
13	-21.45	-13.31	-2.87	9.10	18.42	23.22	21.81	14.62	3.74	-7.82	-18.00	-23.16
14	-21.28	-12.97	-2.48	9.46	18.66	23.27	21.66	14.32	3.36	-8.20	-18.26	-23.22
15	-21.10	-12.63	-2.08	9.82	18.90	23.32	21.50	14.01	2.97	-8.57	-18.52	-23.27
16	-20.91	-12.28	-1.69	10.17	19.13	23.35	21.34	13.69	2.59	-8.94	-18.77	-23.32
17	-20.71	-11.93	-1.29	10.53	19.36	23.39	21.17	13.37	2.20	-9.30	-19.02	-23.36
18	-20.51	-11.58	-0.90	10.88	19.58	23.41	21.00	13.05	1.81	-9.67	-19.26	-23.39
19	-20.30	-11.23	-0.50	11.22	19.80	23.43	20.82	12.73	1.43	-10.03	-19.49	-23.42
20	-20.09	-10.87	-0.10	11.57	20.01	23.44	20.63	12.40	1.04	-10.39	-19.72	-23.43
21	-19.87	-10.51	0.29	11.91	20.22	23.44	20.44	12.07	0.65	-10.75	-19.94	-23.44
22	-19.64	-10.14	0.69	12.25	20.41	23.44	20.25	11.73	0.26	-11.10	-20.16	-23.44
23	-19.41	-9.78	1.08	12.58	20.61	23.43	20.04	11.39	-0.13	-11.45	-20.37	-23.43
24	-19.17	-9.41	1.47	12.91	20.80	23.41	19.84	11.05	-0.52	-11.80	-20.58	-23.42
25	-18.93	-9.04	1.87	13.24	20.98	23.39	19.62	10.71	-0.91	-12.15	-20.77	-23.39
26	-18.68	-8.66	2.26	13.56	21.15	23.36	19.40	10.36	-1.30	-12.49	-20.97	-23.36
27	-18.42	-8.29	2.65	13.88	21.32	23.32	19.18	10.01	-1.69	-12.83	-21.15	-23.32
28	-18.16	-7.91	3.04	14.20	21.48	23.27	18.95	9.66	-2.08	-13.16	-21.33	-23.28
29	-17.89		3.43	14.51	21.64	23.22	18.71	9.31	-2.47	-13.50	-21.50	-23.22
30	-17.62		3.82	14.82	21.79	23.16	18.47	8.95	-2.85	-13.83	-21.67	-23.16
31	-17.34		4.21		21.94		18.23	8.59		-14.15		-23.09

Table 3