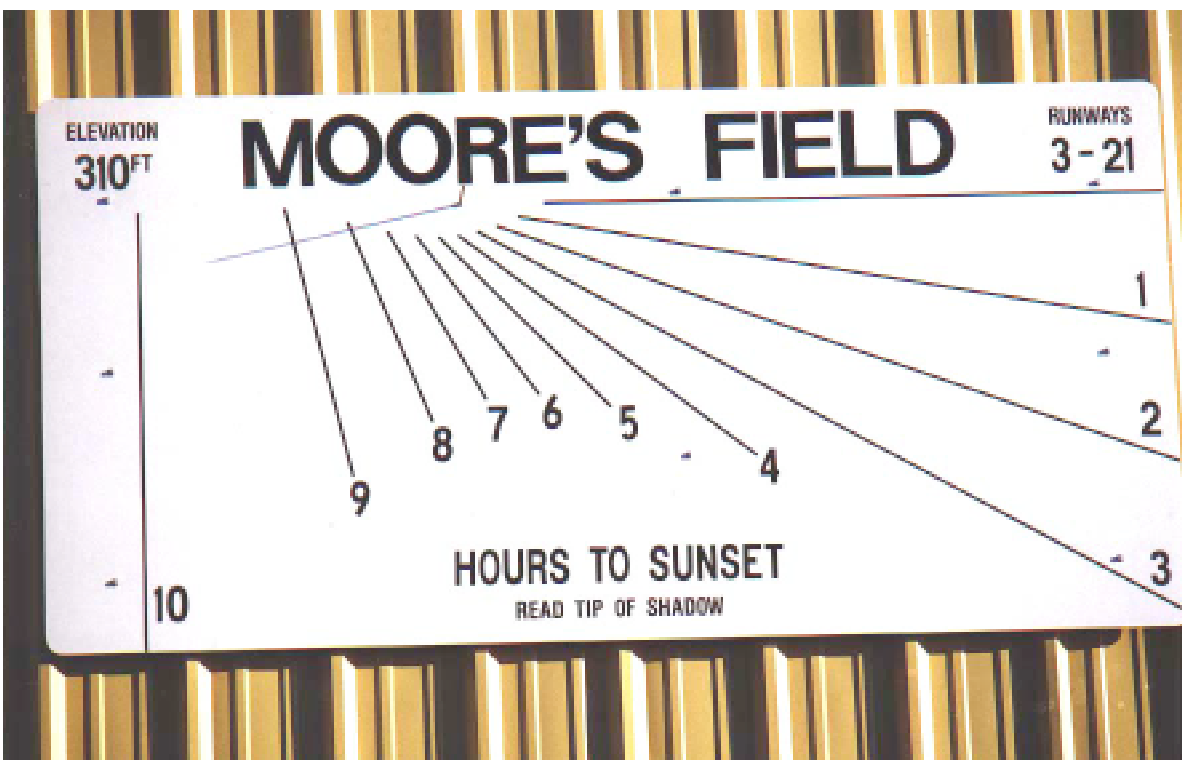
A Design and Construction Manual for Flat Vertical Sundials Which Show Hours To Sunset (Italian Hours Labeled in Countdown Fashion)

by Mac Oglesby



"About 9 hours 40 minutes until sunset"

A Design and Construction Manual for Flat Vertical Sundials Which Show Hours to Sunset (Italian Hours Labeled in Countdown Fashion)

by Mac Oglesby 970820 (revised 040210)

= =
NOTE: This manual originally contained text-based, "do-it-yourself" diagrams. This revision, a PDF file, has the diagram lines drawn. You should ignore any diagram completion instructions which remain.
= =

CONTENTS

Although originally conceived and written for northern latitudes N24 to N66 only, at least some of the material herein is suitable for use in latitudes S66 to N66.

INTRODUCTION

This manual gives detailed instructions for the reader to design and make a sundial which shows Italian hours, sometimes called an "hours to sunset" sundial. Although relatively rare, such sundials have been around for a long time. For thousands of years sundials were an important means of telling time, and since during the last few centuries the science and art of "dialing" has been rigorously examined, it is difficult for anyone now to devise a new sundial design or a new method of realizing an old design. What I've written in these pages consists almost entirely of a restatement, or rearranging of others' ideas and methods. Unfortunately, having accessed many books, articles, and private letters, it's very possible to misremember the source(s) for some of the material.

My first contact with a book on sundials was Albert E. Waugh's, "Sundials, Their Theory and Construction" (Dover), and much of my information stems from that highly recommended source. I am especially indebted to articles and correspondence by William S. Maddux, Frederick W. Sawyer III, and Fer J. de Vries, all of whom have patiently responded to numerous questions. Other dialists worldwide have lent support and assistance, including Robert Terwilliger and Warren Thom. My grateful appreciation goes to everyone who has helped me learn more about dialing.

With this manual, the reader will need additionally only simple tools and a basic scientific calculator, but the design process would be much quicker given access to the computer programs ZONWVLAK and the DIALIST'S COMPANION. Appendix A tells where to obtain these invaluable programs.

Every effort has been taken to ensure the accuracy of the material presented herein, but the author cannot accept responsibility for any damages or inconveniences resulting from use of any of the information in this booklet. Please promptly notify the author of errors. All comments and questions are welcomed.

*** Warning *** Dialing, the involvement with the history, art, and science of sundials, is very intriguing and rewarding, and the reader may find that a little dabbling with sundials leads to a life-long interest.

Good luck with your project. Please send me a photo of the finished sundial. Sunny days and happy dialing,

Mac Oglesby
43 Williston Street
Brattleboro, VT 05301
email: oglesby@sover.net

First, we need to define some terms as used in this manual:

North, South, East, West - true or geographic directions, where your local meridian is a north-south plane including the axis of the Earth and a point (your location), and extends skywards indefinitely. The location of your local meridian is measured as longitude west (+) or east (-) of the Prime Meridian.

Declination - when applied to a vertical wall it means the direction a perpendicular to the wall points relative to south, measured positive west, negative east . A declination (or azimuth) of 15 means the wall faces 15 degrees west of south. (This would be 195 degrees from north.)

Declination - when applied to the sun it means the angular distance north (+) or south (-) of the celestial equator. The declination of the sun varies from -23.44 to +23.44 degrees.

Azimuth - an angle measured along the horizon from south to where the vertical plane including the zenith and the sun meets the horizon, west being positive. An azimuth of -80 means a direction 80 degrees east of south. (This would be 100 degrees from north.)

Local Apparent Time (LT) - is local solar time, as opposed to standard time (ST) or daylight savings time (DT), which is what your clock normally shows. 12:00 noon local solar time is when the sun crosses your local meridian. To convert between Standard Time and Local Apparent Time, refer to Appendix E (2.1 and 2.2)

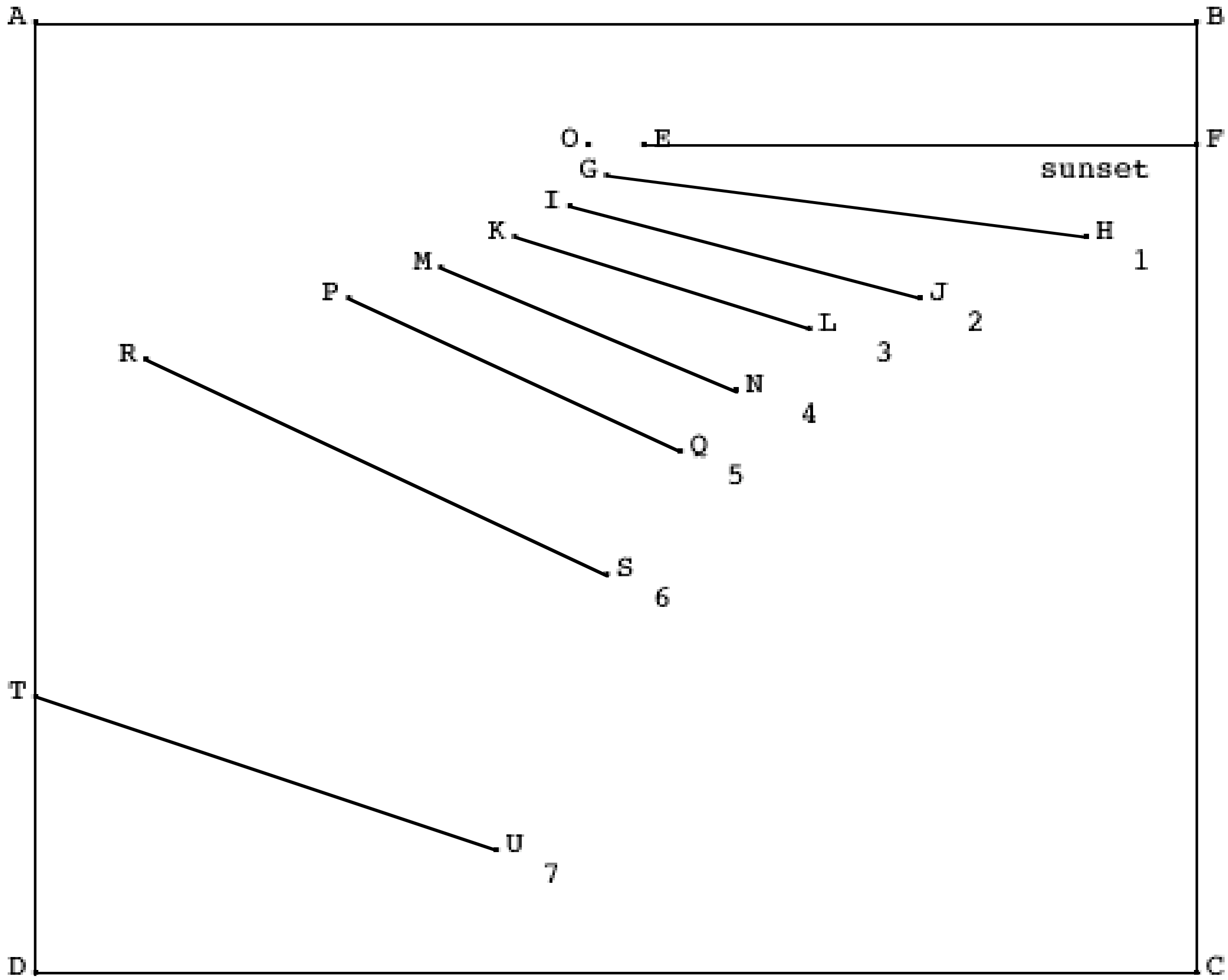
Equation of Time (EQ) - is the accumulated difference between a period of solar days and average, or mean days. The shape of the Earth's orbit and the tilt of its axis create solar days which are unequal in length.

Local Mean Time (LMT) - is what an accurate clock shows if it was accurately set at 12:00 noon local solar time on a day (April 16, June 14, September 1, or December 25) when the equation of time is zero, and then not moved east or west away from that local meridian.

Standard Time (ST) - is local mean time adjusted for the difference in longitude between a local meridian and the standard time meridian for that time zone. Each degree of difference in longitude calls for a 4 minute correction. Normally, our clocks show standard time (or daylight time, which is standard time plus one hour).

For an excellent explanation of Local Mean Time, the Equation of Time, and Standard Time please refer to Chapter 2 of Waugh's book.

DIAGRAM A: Let's see what an hours to sunset sundial looks like at a certain latitude and orientation. Scale is about 1:4.



To complete this diagram, draw the following line segments:

1. draw rectangle ABCD
2. draw sunset hour line EF
3. draw hour lines GH, IJ, KL, MN, PQ, RS, and TU

The completed diagram shows, approximately, the appearance of an hours to sunset sundial designed for latitude N39.69 and facing true southwest (45 degrees west of true south). The hour lines for a dial at another latitude or declination would be very different. The gnomon is a pointed post 100 mm long set perpendicular to the dial at the point labeled O. The outside border shown is about 24 by 24 in. The extreme tip of the gnomon's shadow (not shown) indicates the time to sunset.

Quoting from the Dialist's Companion, "Babylonian and Italian hours are from two of the earliest systems of recording time to use equal hours. The Babylonian system divides the day into 24 hours beginning at sunrise and ending with the following sunrise. The Italian system also divides the day into 24 hours but begins counting at sunset and ends with the following sunset..."

An hours to sunset sundial shows Italian hours but labeled in countdown fashion. These dials may be drawn on any surface, but this manual deals only with dials drawn on vertical flat surfaces.

An outstanding feature of this type of sundial is that no corrections are necessary for longitude displacement, equation of time, or daylight savings time. This sundial does what a sundial does best - it tells us what the sun is doing, not what our clocks are doing.

Although the sundials mentioned in this manual were designed and made for use at airports, there are many other likely places where such a sundial would be appropriate. Think about an hours to sunset sundial at a boat marina, or a ski resort, or a playground.

The Hours to Sunset Sundial at Moore's Field

Moore's Field is a grass airstrip located a few miles north of Brattleboro, Vermont. Since there are many light aircraft based there which should be back by dark, it seemed logical to make and install an hours to sunset sundial.

The sundial is a flat, vertical 4' by 8' sheet of white painted MDB (maximum density board) with black vinyl lines, letters, and numerals. The gnomon, or shadow-casting device, is a 5 mm diameter stainless steel tapered post 160 mm in length set perpendicular to the face of the dial. The sundial faces 8.4 degrees west of south (true, not magnetic) at latitude N42.93 degrees. Installation was at least as time consuming as the sundial construction, for the hanger wall wasn't stiff, flat, or plumb. Shims and stiffening members were necessary so that the sundial would be flat and vertical. Eight stainless bolts hold the sundial to the building.

The computation of the hour lines was done with a suite of computer programs called Zonwvlak (plane sundial) by their author, Dutch dialist Fer J. de Vries. Zonwvlak provided (x,y) coordinates for each end of each hour line, with the origin (0,0) being the foot of the gnomon. Zonwvlak also provided a drawing of the hour line layout, which was very useful for planning the sundial. Appendix A tells how to obtain Zonwvlak, now named zw2000.

I've been asked to supply plans for this kind of sundial, and I wish it were that simple. But hours to sunset sundials are site specific and must be computed for a particular latitude and orientation. Remembering that your sundial's accuracy depends upon the careful installation of a carefully crafted dial using coordinates carefully computed from accurate data, to design your dial you'll need to collect certain information

First, decide on a location where the sundial will be sunlit at the end of the day. Second, find the sundial's exact latitude and longitude. Third, determine the direction the sundial will face relative to true (not magnetic) south. A west or southwest facing direction is best; south is OK. If your sundial mounts on a wall, you'll need to determine the wall's declination. For preliminary planning you can use a magnetic compass, remembering to adjust for local magnetic variation, but to compute an accurate sundial you'll need its declination to the nearest degree. Consider installing your sundial on posts, or otherwise making its declination adjustable. Then you can compute for any appropriate and convenient declination and install the dial to match it. Finally, you need to decide on the overall size of your sundial and the length of the perpendicular gnomon. A gnomon length of about one-eighth of the width of the sundial is a reasonable place to start. The dial size and the gnomon length are directly related. Hence, you can compute your x,y coordinates using a gnomon length of 100 (think millimeters) and easily change everything. If you later find that a gnomon length of 260 means a better layout, simply multiply each x and y value by 2.6.

To accurately determine a wall's declination, use Appendix E of this manual. Alternatively, a surveyor can determine in which direction the wall faces relative to true south.

You can use Zonwvlak (zw2000) to compute the x,y coordinates for each end of each hour line. Without Zonwvlak, you can compute the coordinates using Chapter 3 and/or Appendix D of this manual.

Use painted plywood, maximum density board, or stiffened plastic for your dial. You can put the desired letters, numbers, and hour lines on the dial with paint, or apply vinyl characters (from an art supply shop) and vinyl striping (from an auto parts store). Because confusing shadows may result, avoid raised hour lines. The gnomon is a pointed post, firmly fixed to stay perpendicular.

The proper installation of your sundial is vital to ensure continuing accuracy. The dial must fastened so as to remain flat, vertical, and properly orientated.

3.1.0 As stated previously, Italian hours are used in a time system which divides each day into 24 equal hours, beginning and ending at sunset. If a sundial has Italian hour lines labeled in countdown fashion, it shows hours remaining until sunset. Such a sundial needs no corrections for longitude, equation of time, or daylight savings time.

Our task, to compute the Italian hour lines for a vertical sundial of known orientation at a given latitude, is simplified because the hour lines we need will all be straight lines. Stepping backwards from the time of sunset, we'll compute the coordinates for each end of each required hour line. To plot these coordinates, we'll use a normal x,y coordinate system with the x-axis horizontal and the y-axis vertical. Facing the dial, positive y is up, positive x is to the right of the origin, the foot of the perpendicular gnomon. The shadow of the perpendicular gnomon's extreme tip indicates the hours to sunset.

To determine the location of the Italian hour lines on our sundial, we need to find the coordinates for the shadow of the gnomon's tip at one-hour intervals from sunset when the sun is at its greatest angular distance above and below the celestial equator. The angular distance of the sun from the celestial equator is called the solar declination and varies from -23.44 on December 21 to +23.44 on June 21. For a given hour line on a direct south facing vertical dial in the Northern Hemisphere, the shadow of the gnomon's tip will lie closest to gnomon's foot when the sun is lowest in the sky, at solar declination -23.44.

We aren't limited to one-hour intervals for the "hour" lines, but can produce coordinates for half-hour lines, or even smaller intervals if we wish. To avoid a cluttered dial, one-hour intervals are recommended. Let the viewer estimate the "between" times.

Sometimes a given solar declination and hour angle combination will not compute usable coordinates, either because the sun doesn't illuminate the dial or the shadow falls beyond the boundary of the dial. Different solar declinations may be chosen until a usable shadow results. If for a certain hour line the end-point nearest the gnomon is known and you are sure the hour line extends beyond the boundary of the dial, you can draw the hour line if you can determine its slope. Example C includes an explanation of how to find the slope of a line.

It is recommended that your x,y coordinates be computed for a gnomon length of 100.0 (think millimeters). Since the coordinates are directly related to the size of the gnomon, you can easily scale up or down. For example, if you later decided to use a gnomon length of 75, just multiply each x and y value by 0.75.

3.2.0 In the steps and formulas which follow,

HA = hour angle of the sun before or after local apparent noon, calculated at 15 degrees per hour with afternoon positive, morning negative. At sunset, HA = Hs; at 9 am, HA = - 45; at 6 pm, HA = 90

Hs = time of sunset as an hour angle from noon; each hour = 15 degrees

Lat = latitude of the sundial's location

DC = the declination of the sun relative to the celestial equator

dZD = the difference between the azimuth of the sun and the declination of the dial.

Z = azimuth of the sun, measured as an angle along the horizon from true south, with west positive, to a vertical from the sun

D = the declination of the dial, measured from true south, with west positive

ALT = altitude of the sun measured vertically from the horizon

g = length of the gnomon, needle type, perpendicular to dial

* means multiplication / means division.

3.2.1 These steps assume the latitude and declination of the dial are known and gnomon length has been decided upon.

1. Choose solar declination DC; (normal start is -23.44). Ideally, we'd compute all of the hour line end-points at -23.44 and then at +23.44. In fact, we may need to use some intermediate solar declinations to get usable coordinates. Avoid using DC = 0

2. Using chosen solar declination, compute hour angle Hs for sunset.

3. Compute hour angle HA for required sundial hour line.

4. Compute the sun's altitude ALT at chosen HA and DC.

5. If sun is above the horizon, continue. Else, choose a different DC and return to step 2.

6. Compute the sun’s azimuth Z at the desired HA and DC.

7. If dial face is illuminated by sun, continue. Else, choose a different DC and return to step 2.

8. Compute value of dZD.

9. Compute x,y, coordinates. If point x,y is on dial, continue. Else, choose another value of DC and go to step 2.

10. Continue computing hour line end-points. Pick another hour before sunset line and return to step 1.

3.2.2 The formulas:

To draw the hour lines, we first need to know the time of sunset. That time, as an angle from noon, is given by:

(1) Cos Hs = - Tan Lat * Tan DC

This angle may be divided by 15 to convert to hours, should you wish. Remember, the result is local apparent time, not clock time.

The altitude ALT of the sun is given by:

(2) Sin ALT = Sin DC * Sin Lat + Cos DC * Cos Lat * Cos HA

The dial is illuminated only if the sun is above the horizon; i.e., the altitude of the sun is greater than zero.

The azimuth Z of the sun is given by:

(3a) N = Sin Lat * Cos HA - Cos Lat * Tan DC

(3b) Tan Z = Sin HA / N

(3c) (if N < 0, Z = Z + 180)

After we determine the value of Z, we can compare Z with D to see if the dial is illuminated by the sun at time HA, and if so, continue our computations. The sun is behind the dial if its azimuth differs from the dial’s declination by more than 90 degrees.

```
(4)     dZD = Z - D

(5)     x = g * Tan dZD

(6)     y = - g * Tan ALT / Cos dZD
```

3.3.0 Some worked problems

EXAMPLE A:

The Moore's Field sundial is at latitude N42.93 and declines 8.4 degrees west of south. The 1219 mm by 2438 mm dial/sign board has a perpendicular gnomon 160 mm long, with its base (x=0,y=0) located 250 mm from the dial's upper edge and 900 mm from the dial's west edge. This means the effective dial width is 969 mm. The dial has 900 mm of space west of the gnomon, (or to the left as you face the dial), and 1538 mm east, or to the right).

We compute the sunset hour angle Hs on December 21 when the sun's declination is -23.44.

```
(1)  Cos Hs = - Tan Lat * Tan DC
            = - 0.930233556 * - 0.433567758
            = 0.403319277
         Hs = 66.21415275
```

We compute the azimuth of the sun Z and the sun's altitude ALT at HA = Hs.

```
(2)  Sin ALT = Sin DC * Sin Lat + Cos DC * Cos Lat * Cos HA
         ALT = 0   (0 altitude of the sun for sunset and sunrise)


(3a)  N = Sin Lat * Cos HA - Cos Lat * Tan DC
        = 0.681104334 * 0.403319277 - 0.732186373 * - 0.433567758
        = 0.274702508 - - 0.317452405
        = 0.592154913

(3b)  Tan Z = Sin HA / N
            = 0.91505932 / 0.592154913
            = 1.54530394
          Z = 57.09221028

(3c)  (N is positive, so we don't change Z)
```

Since this difference between the declination of the dial and the sun's azimuth is about 49 degrees, this hour line end-point will exist on the dial face. In practice, the sun's azimuth should not differ from the dial's declination by more than about 87 or 88 degrees, depending on the gnomon's length and placement.

```
(4)     dZD = Z - D = 57.09221028 - 8.4 = 48.69221028
```

We compute the values of coordinates x and y, using g = 160.

```
(5)     x = g * Tan dZD
          = 160 * 1.137964051
          = 182.07

(6)     y = - g * Tan ALT / Cos dZD
          = - 160 * 0 / 0.660132778
          = 0     (for sunset hour line, y = 0)
```

One end of the sunset hour line will be at (x=182.1, y=0).

To find the other end of the sunset hour line, we compute the sun's azimuth Z on June 21, when the sun's declination is 23.44.

```
     Cos Hs = - Tan Lat * Tan DC
            = - 0.930233556 * 0.433567758
            = - 0.403319277
         Hs = 113.7858472
```

The altitude of the sun at sunset is 0.

We compute the azimuth of the sun Z at HA = Hs.

First, we compute a temporary value, N.

```
     N = (Sin Lat * Cos HA - Cos Lat * Tan DC)
       = (0.681104334 * - 0.403319277 - 0.732186373 * 0.433567758)
       = - 0.592154913

     Tan Z = Sin HA / N
           = 0.91505932 / - 0.592154913
           = - 1.54530394
         Z = - 57.09221028
```

```
Since N < 0, Z = Z + 180           Z = - 57.09221028 + 180

     Z = 122.9077897
```

We notice that the sun's azimuth is more than 90 degrees from the dial's declination, so the sun doesn't illuminate this dial at sunset on June 21, when the sun's declination is 23.44. We could find the date and declination of the sun which would produce a shadow from the gnomon's tip falling just at the eastern edge of the dial, but why bother. The sunset line is horizontal, with one end at (x=182.1, y=0), and runs horizontally to the eastern edge of the dial.

EXAMPLE B: Let's do another hour line for the Moore's Field dial, 6 hours before sunset. Gnomon = 160; declination = 8.4.

On December 21 the sun's declination is -23.44 and the hour angle to sunset Hs is 66.21415275. Each hour represents 15 degrees, so at 6 hours before Hs the sun will be at hour angle Hs - 90 degrees. HA = 66.21415275 - 90 = - 23.78584725

```
     Sin ALT = Sin DC * Sin Lat + Cos DC * Cos Lat * Cos HA
             = - 0.397788507 * 0.681104334
                      + 0.91747714 * 0.732186373 * 0.91505932
             = - 0.270935476 + 0.614704147
             = 0.34376871
         ALT = 20.10664903
```

The sun is above the horizon.

Next, we compute the sun's azimuth Z.

```
     N = (Sin Lat * Cos HA - Cos Lat * Tan DC)
       = (0.681104334 * 0.91505932 - 0.732186373 * - 0.433567758)
       = 0.940703274

     Tan Z = Sin HA / N
           = - 0.403319277 / 0.940703274
           = - 0.428742292
         Z = - 23.20686066

     (If N < 0, add 180 to Z).

     Z = - 23.20686066         (HA and Z must agree in sign.)
```

The difference between Z and D is about 32 degrees, so this end-point should be on the dial.

= =
NOTE: If it isn't obvious if Z and D are within 90 degrees of each other, draw a diagram. For example, if D=180 and Z=-110, the formula dZD=Z-D suggests that dZD=-110-180 =-290. A sketch shows that a dial declining 180 has a gnomon which points due north which is only 70 degrees from the sun's azimuth of -110. For our computations, -290 is the mathematical equivalent of 70.
= =

```
dZD = Z - D = -31.60686066

x = g * Tan dZD
  = 160 * - 0.615369177
  = - 98.5


y = - g * Tan ALT / Cos dZD
  = - 160 * 0.366079627 / 0.851635468
  = - 68.8
```

Therefore, on December 21 one end of the 6 hours before sunset line is located at (-x=98.5, y=-68.8)

For the other end of the 6 hours before sunset line, let DC = 23.44. Hs = 113.7858472. HA = 90 degrees (6 hours) earlier or 23.7858472.

```
Sin ALT = Sin DC * Sin Lat + Cos DC * Cos Lat * Cos HA
        = 0.270935476 + 0.614704148
        = 0.885639624
    ALT = 62.33033676
```

Azimuth computation:

```
N = Sin Lat * Cos HA - Cos Lat * Tan DC
  = 0.681104334 * 0.91505932 - 0.732186373 * 0.433567758
  = 0.623250869 - 0.317452405
  = 0.305798464

Tan Z = Sin HA / N
      = 1.318905501
    Z = 52.83043436
```

(If N < 0, add 180 to Z, and signs of N and Z must agree.)

The difference between Z and D is about 44 degrees, so this end-point should exist on the dial.

```
dZD = Z - D = 44.43043436


x = g * Tan dZD
  = 160 * 0.980313459
  = 156.85
```

y = - g * Tan ALT / Cos dZD
= - 160 * 1.907172188 / 0.714100931
= - 427.3

The other end-point for the 6 hours before sunset line will be at (x = 156.85, y= - 427.3).

EXAMPLE C: Let's do the 1 hour before sunset line for the Moore's Field sundial. Gnomon = 160; declination = 8.4.

On December 21 the sun's declination is -23.44 and Hs = 66.21415275. HA = Hs - 15 = 51.21415275.

ALT = 8.619117007

N = 0.744103847

Z = 46.33058093

dZD = 46.33058093 - 8.4 = 37.93058093

x = 124.7 y = -30.7

On June 21 the sun's declination is 23.44 and Hs = 113.7858472. HA = Hs - 15 = 98.7858472.

N = -0.421485456

Z = 113.0977547 Since this is more than 90 degrees from the declination of the dial, the sun is behind the plane of the dial at solar declination +23.44 one hour before sunset. A solar declination of -23.44 gave us one point on the dial, but we need another. If we visualize the plane of the dial relative to the vertical plane which includes the sun and the gnomon's tip, we realize the one hour before sunset line must extend indefinitely eastward, sloping downward to the edge of the dial. All we need is a second point towards which to draw the needed line.

Let's try a solar declination of 8.25. Hs = 97.75153328 and HA = Hs - 15 = 82.75153328.

ALT = 10.90371736

N = -0.020225388

Z = 91.16800315 (This is acceptable, being less than 90 degrees from the declination of the dial.)

dZD = 82.76800315

x = 1260.9 y = -244.8

If these coordinates are on the dial, draw this hour line from coordinates (124.7,-30.8) through coordinates (1260.9,-244.8) to the dial's edge. If the second coordinates are too far off the dial's face to plot, we can calculate the line's slope and then determine a point on the dial.

Slope = change in y divided by change in x.

Slope = (y2 - y1) / (x2 - x1)

Slope = (-244.8 - -30.8) / (1260.9 - 124.7) = -0.1883471

We select any convenient x3 coordinate (say 800) and calculate the y3 coordinate. The x value changes by 800 - 124.7 or 675.3. Since slope equals change in y divided by change in x, the change in y equals slope multiplied by change in x. Change in y equals -0.1883471 * 675.3 or -127.2, and y3 equals -127.2 + -30.8, or -158.0. The desired coordinates are (800,-158.0).

EXAMPLE D: Now that we've warmed up the calculator and seen how the methodology works, let's compute all of the hour lines for the vertical dial pictured in DIAGRAM A. The latitude is N39.69 and the declination is 45. The gnomon length g = 100.

This time we'll try to be more organized. Take a piece of lined notebook paper and draw lines to make 8 columns. From left to right, label the columns, with HL meaning Hour Line before sunset:

HL	DC	Hs	HA	ALT	Z	dZD	x, y
0	-23.44	68.91	68.91	0	58.87	13.87	24.7, 0
0	+23.44	111.09	111.09	0	121.13	76.13	405.0, 0
1	-23.44	68.91	53.91	9.31	48.70	3.70	6.5, -16.4
1	+23.44	111.09	96.09	10.32	111.98	66.98	235.4, -46.6
2	-23.44	68.91	38.91	17.18	37.10	-7.90	-13.9, -31.2
2	+23.44	111.09	81.09	21.31	103.36	58.36	162.3, -74.4
3	-23.44	68.91	23.91	23.04	23.83	-21.17	-38.7, -45.6
3	+23.44	111.09	66.09	32.70	94.68	49.68	117.8, -99.2
4	-23.44	68.91	8.91	26.32	9.12	-35.88	-72.3, -61.1
4	+23.44	111.09	51.09	44.23	85.04	40.04	84.0, -127.1

HL	DC	Hs	HA	ALT	Z	dZD	x, y
5	-23.44	68.91	-6.09	26.61	-6.25	-51.25	-124.6, -80.1
5	+23.44	111.09	36.09	55.54	72.79	27.79	52.7, -164.7
6	-23.44	68.91	-21.09	23.87	-21.16	-66.16	-226.3, -109.4
6	+23.44	111.09	21.09	65.89	53.92	8.92	15.7, -226.2
7	-23.44	68.91	-36.09	18.45	-34.73	-79.73	-551.9, -187.1
7	+23.44	111.09	6.09	72.95	19.39	-25.61	-47.9, -361.7
8	-23.44	68.91	-51.09	10.92	-46.64	-91.64	(sun behind dial)
8	+23.44	111.09	-8.91	72.09	-27.52	-72.52	-317.5, -1030.0
8	-10.00	81.59	-38.41	28.87	-44.33	-89.33	-8538.2, -4708.5

(The line for 8 hours before sunset would either be omitted from this dial, or point (-317.5,-1030) would be plotted, the slope computed, and the line drawn to the edge of the dial. Personally, I would certainly omit 8, and maybe 7 also.)

I strongly recommend that you plot the coordinates in millimeters. Remember that you can move the hour lines farther apart or closer together by taking a larger (or smaller) value for g, the length of the gnomon. As stated previously, if your coordinates were computed for g = 100, but you want to use g = 260, multiply each value of x and y by 2.6.

APPENDIX A Selected References

Books:

Mayall, R. Newton, and Mayall, Margaret W. "Sundials: How to Know, Use, and Make Them." 2nd Edition, Cambridge, MA, 1973

Rohr, Rene R. J. "Sundials: History, Theory, and Practice." New York: Dover, 1996 This book contains several pages about sundials which have Italian hours.

Waugh, Albert E. "Sundials, Their Theory and Construction." New York: Dover, 1973.

Internet resources:

Of the many web pages available, I am most familiar with the NASS site, Robert Terwilliger, webmaster. Go to <http://sundials.org> for an amazing assortment of items. Click on Links, then Software for sundial computer programs, including:

Dialist's Companion, The. A computer program written by Robert Terwilliger and Frederick W. Sawyer III which provides essential data, including Equation of Time, Declination of Sun, Local Apparent Noon, more.

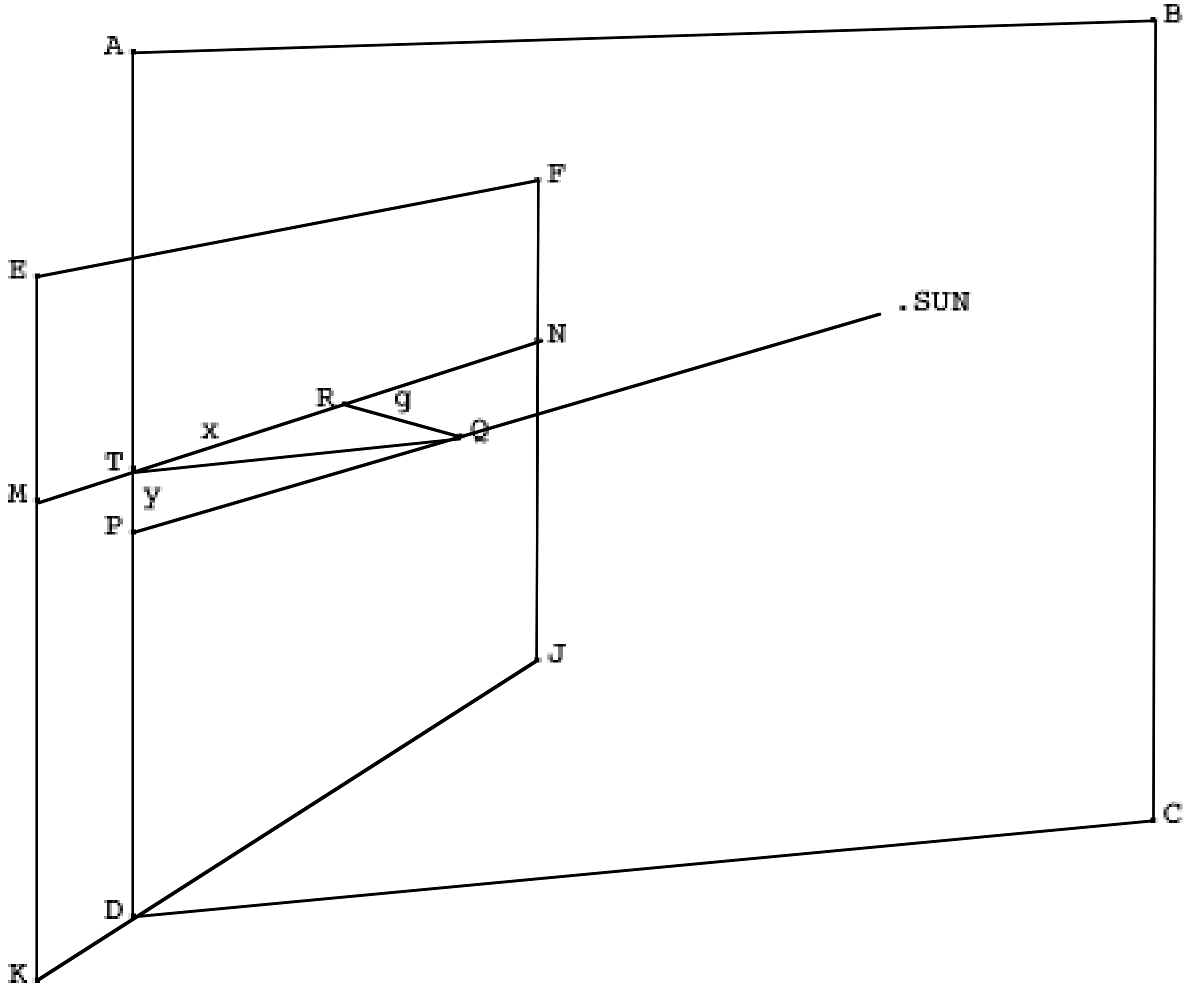
Zonwvlak (zw2000). A suite of computer programs written by Fer J. de Vries, which generates data for manual, computer screen, or printer plots for a variety of sundial types on a plane.

Also from the NASS home page:

- click on Links, then General for Daniel Roth's Sundial Links. Or, go to <http://www.infraroth.de/slinks.html>
- for information on national sundial societies, click on Links, then Societies
- for a great source of basic sundial information, click on F.A.Q.

I especially recommend The NASS Repository Disk, which not only contains all back issues of The Compendium, but also an amazing collection of sundial material, and several computer programs. For details on this CD, from the NASS home page, click on Publications.

APPENDIX B Derivation of Formulas for x,y Coordinates.
(This method, for a vertical dial, was suggested by Wm Maddux. Additional suggestions by Fer de Vries.)



To complete this drawing,

1. draw lines AB, BC, CD, and AD
2. draw lines EF, FJ, JK, and EK
3. draw line MN through point T and point R
4. draw line PQ and extend it to point SUN
5. draw lines QR and QT

The drawing shows a flat, vertical sundial intersected by a vertical plane which passes through the sun and the gnomon's tip. The gnomon's shadow, which would stretch from R to P, has been deliberately left off of this drawing.

Flat vertical sundial EFJK has pointed post gnomon QR of length g perpendicular to the dial at R. Horizontal line MN passes through R. If the vertical plane ABCD which passes through the sun (considered to be a point) and the gnomon's tip Q intersects the dial, vertical line PT is determined. The gnomon's shadow is omitted in this drawing, but would run from R to P, where P marks the shadow of Q. If R is the origin of an x,y coordinate system with x positive to the right (as you look at the dial) along horizontal MN, and y positive toward the zenith, the x,y coordinates of P are the lengths of RT and PT respectively. Angle RQT equals the difference (dZD) between the azimuth angles of the sun and the declination of the dial. Angle PQT equals the altitude (ALT) of the sun.

Since triangles QRT and QTP are right triangles,

Tan dZD = RT / g
= x / g, so, x = g * Tan dZD.

Tan ALT = PT / QT
= y / QT, so, y = Tan ALT * QT

But Cos dZD = g / QT, so, QT = g / Cos dZD.

Thus we have our x and y values:

x = g * Tan dZD

y = - g * Tan ALT / Cos dZD

If our orientation is looking straight out from the dial's face in the direction the gnomon points, and if we assume the shadow of the gnomon falls upon the dial's face, then at that moment the sign of x will be negative if the gnomon is pointing to the right of the sun, positive if the gnomon is pointing left of the sun. The value of y is always zero or negative.

APPENDIX C What About Horizontal Hours to Sunset Sundials?

Certainly it's possible to design and construct a horizontal sundial which shows hours to sunset. A horizontal dial has some advantages: the dial is illuminated by the sun all day long rather than the 12 hour limit for a vertical dial, one doesn't have to determine the declination of a wall, and it's easier to compute the hour lines. On the negative side, a horizontal dial may not offer the excellent visibility of a wall-mounted dial, and the thin, pointed gnomon may present a hazard unless the dial is on a pedestal.

To compute the hour lines for a horizontal hours to sunset dial, follow the formulas from Chapter 3 with these changes: the dial will always be lit by the sun if the sun's altitude is greater than zero, there is no need to compute a value for dZD, and the formulas for coordinates x,y are different.

Formulas for x,y coordinates for horizontal hours to sunset dial:

x = g * Sin Z / Tan ALT

y = g * Cos Z / Tan ALT

The x-axis runs east-west, with positive east, and the y-axis is true north-south, with positive to the north. The origin (0,0) is the foot of the perpendicular gnomon. The sundial must be installed with the y-axis accurately aligned to the local north-south meridian.

APPENDIX D   Spreadsheet for Calculating x,y Coordinates.
(Calculates x,y, coordinates for Italian hour lines for a vertical or horizontal sundial in latitudes 66 south to 66 north (-66 to +66)).

How to re-assemble the spreadsheet, which is presented in three parts. Part 1 shows columns A and B. Part 2 shows columns C, D, and E, where E shows computed values. Part 3 also shows columns C, D, and E, but E displays the formulas. The only purpose of columns A and D is to facilitate alignment of the rows. Column B identifies what is happening. Column E is where it happens.

Part 1 of spreadsheet:

```
A1   Compute x,y for vertical or horizontal sundial showing hours
       to sunset (Italian hours labeled in countdown fashion)
A2             Mac Oglesby  970515    (file name:   Sprdsht970530)
A3   Latitude   (Lat)        Input -66 (south) through 66 (north)
A4   Sundial's Inclination:    0 =  for horizontal dial;
       90 = vertical dial    (other inputs invalid)
A5   Sundial's Declination   (D)     Input -180 to 180;  0=south,
       -90=east, 90=west, 180 or -180=north
A6   Gnomon length   (g)     Recommended input:   100  (think
       millimeters)
A7   Solar Declination   (DC)          Input -23.44 through 23.44
A8   Hours before sunset   (Hb)      Input 0 through 23;  other
       inputs considered 0
A9        Convert degrees to radians:  DR=3.14159265/180
A10       Convert radians to degrees:  RD=180/3.14159265
A11  Hour angle at sunset:  Cos Hs = - Tan Lat * Tan DC
A12  Hour angle before sunset:  HA = Hs - (Hb * 15)
A13  Azimuth Z of sun:   N = Sin Lat * Cos HA - Cos Lat * Tan DC
A14       Tan Z = Sin abs(HA) / N
A15       Add 180 to Z if n is negative
A16       Azimuth and HA must agree in sign
A17  Altitude of sun:  Sin ALT = Sin DC * Sin Lat +
       Cos DC * Cos Lat * Cos HA
A18    Is dial lit by sun?   (Is ALT = > 0, and, if a vertical
       dial, is Z within 90 degrees of D?)
A19      IF(IN=90,IF(SIGN(Z)=SIGN(D),IF(ABS(ABS(Z)-
       ABS(D))<90,"+","---"),"--"),"-")
A20      IF(AND(IN=90,E19<>"+"),IF(SIGN(D)<>SIGN(Z),IF(ABS(ABS(D)+
       ABS(Z))<90,"+","---"),"--"),"-")
A21      IF(AND(IN=90,E19<>"+",E20<>"+"),IF(SIGN(D)<>
       SIGN(Z),IF(360-ABS(D)-ABS(Z)<90,"+","---"),"--"),"-")
A22      IF(AND(IN=90,E18="+"),IF(AND(E19<>"+",E20<>"+",E21<>"+"),
       "Dial Unlit",Z-D),"-")
A23  Vertical dial:  x = g * Tan dZD
A24  Vertical dial:  y = -g * Tan ALT / Cos dZD
A25       Horizontal dial:  x = g * Sin Z / Tan ALT
A26       Horizontal dial:  y = g * Cos Z / Tan ALT
```

Part 2 of spreadsheet:

```
     D1
     D2
Lat  D3   20
IN   D4   90
D    D5   0
g    D6   100
DC   D7   -0.88
Hb   D8   0
     D9   0.0174532925
     D10  57.2957795785523
Hs   D11  89.6796794407721
HA   D12  89.6796794407721
     D13  0.016345882548621
     D14  89.0635187851667
     D15  89.0635187851667
Z    D16  89.0635187851667
ALT  D17  -8.3716687496853e-18
     D18  +
     D19  --
     D20  +
     D21   -
dZD  D22  89.0635187851667
x    D23  6117.65298925508
y    D24  0.00000000000000089398916454189
x    D25  -
y    D26  -
```

Part 3 of spreadsheet:

```
     D1
     D2
Lat  D3   20
IN   D4   90
D    D5   0
g    D6   100
DC   D7   -0.88
Hb   D8   0
     D9   =3.14159265/180
     D10  =180/3.14159265
Hs   D11  =ACOS(-TAN(E3*E9)*TAN(E7*E9))*E10
HA   D12  =IF(AND(E8>=0,E8<24),E11-E8*15,E11)
     D13  =SIN(E3*E9)*COS(E12*E9)-COS(E3*E9)*TAN(E7*E9)
     D14  =ATAN(SIN(ABS(E12)*E9)/E13)*E10
     D15  =IF(E13<0,E14+180,E14)
Z    D16  =IF(SIGN(E12)=SIGN(E15),E15,E15*-1)
ALT  D17  =ASIN(SIN(E7*E9)*SIN(E3*E9)+
            COS(E7*E9)*COS(E3*E9)*COS(E12*E9))*E10
     D18  =IF(E17>-0.00001,"+","No Sun")
     D19  =IF(E4=90,IF(SIGN(E16)=SIGN(E5),IF(ABS(ABS(E16)-
            ABS(E5))<90,"+","---"),"--"),"-")
     D20  =IF(AND(E4=90,E19<>"+"),IF(SIGN(E5)<>SIGN(E16),IF
```

```
             (ABS(ABS(E5)+ABS(E16))<90,"+","---"),"--"),"-")
     D21  =IF(AND(E4=90,E19<>"+",E20<>"+"),IF(SIGN(E5)<>SIGN(E16),
             IF(360-ABS(E5)-ABS(E16)<90,"+","---"),"--")," -")
dZD  D22  =IF(AND(E4=90,E18="+"),IF(AND(E19<>"+",E20<>"+",E21<>
             "+"),"Dial Unlit",E16-E5),"-")
x    D23  =IF(AND(E4=90,E18="+",ISNUMBER(E22)),E6*TAN(E22*E9),"-")
y    D24  =IF(AND(E4=90,E18="+",ISNUMBER(E22)),
             -E6*TAN(E9*E17)/COS(E9*E22),"-")
x    D25  =IF(AND(E4=0,E18="+"),E6*SIN(E16*E9)/TAN(E17*E9),"-")
y    D26  =IF(AND(E4=0,E18="+"),E6*COS(E16*E9)/TAN(E17*E9),"-")
```

This spreadsheet was created using ClarisWorks 4.0 (Macintosh). It has proved to be useful, especially when checking calculator results, but the author does not guarantee its accuracy, nor that it is free from errors.

(Note: The reader is again urged to obtain a copy of Albert Waugh's excellent book, "Sundials, Their Theory and Construction" (Dover - recently $7.95), which explains this process and is a fine source of basic information about time and sundials.)

To find the declination of a vertical wall we'll use these steps:

1) Measure the angle W along the horizon between the sun and a perpendicular to the face of the wall.

2) Compute, or look up the azimuth Z of the sun at the time of measuring angle W.

3) Use the angles W and Z to determine the declination of the wall.

To accomplish this, you need to know the sundial's latitude and longitude. Also, you will need an accurate timepiece, a plumb line, a ruler, a large square, a level, and a scientific calculator capable of basic trig functions.

= =
NOTE: If you determine the angle W between the sun and the wall's perpendicular exactly at noon Local Apparent Time, your task is greatly simplified, for at local noon the sun is due south and the angle measured is the wall's declination. To use this short cut, first calculate Standard Time for local noon (see section E.2.2), then follow the steps in sections E.1.0 through E.1.5.
= =

Step 1. Find angle W between sun and the wall's perpendicular.

E.1.0 If you can't mark directly on the wall, place masking tape, drawing paper, or a flat board firmly against it. If the wall isn't flat, or if its surface is uneven, orient the board as the sundial will be installed.

E.1.1 Hang a plumb line about 12 inches away from the wall. See Diagram #1. If wind causes the line to sway, let the plumb weight hang inside an empty pail. If needed, fill the pail with water.

DIAGRAM #1: A view from above

```
                  P. Plumb line


_Wall___________________________________
```

E.1.2   Using a large square (a right angle plywood corner will do), mark M where a horizontal perpendicular through the center of the plumb line P meets the wall. Draw a horizontal through M.

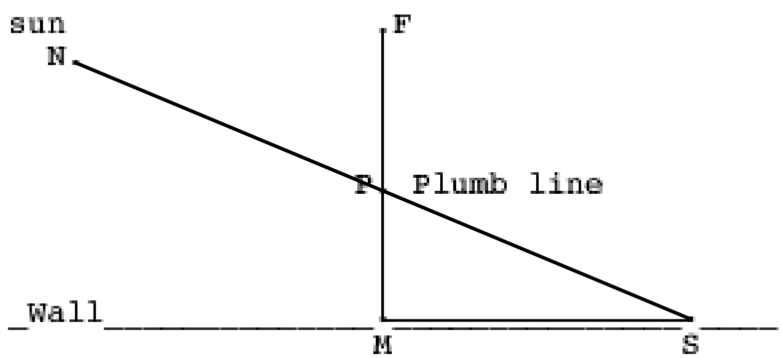
E.1.3   Recording the precise time (this is vital), carefully mark where the center of the plumb line's shadow S crosses the horizontal drawn through M. See Diagram #2.

DIAGRAM #2: A view looking at the surface of the wall

```
_________________________________________
|                                       |
| horizontal line                       |
|-----------------+---------------+-----|
|                 M               S  Shadow of plumb line
|   Perpendicular from plumb line       |
_________________________________________
```

3.1.4   Measure the precise distance MP from the center of the plumb line to the wall and the precise distance MS.

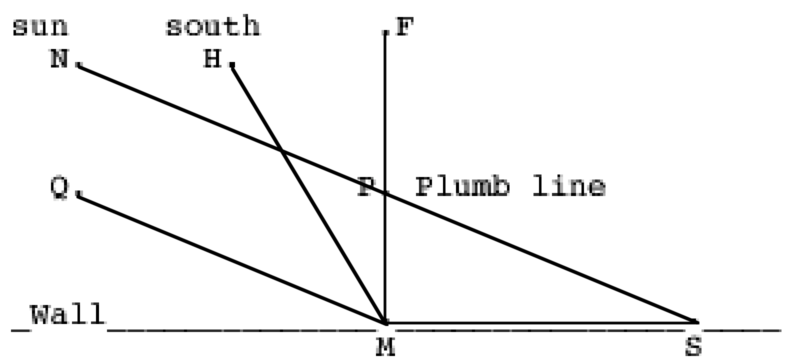
DIAGRAM #3: A view from above.



E.1.5 Angle NPF is the angle W between the sun and the perpendicular to the wall. Angle MPS equals angle NPF, so

Tan W = MS divided by MP or, W = Arctan (MS / MP)

DIAGRAM #4: A view from above. To complete this diagram, draw the lines as for diagram 3. Then draw a line from south, H, to M. Assuming the sun's rays to be parallel upon arriving at the wall, draw a line from M to Q, parallel to PS.



E.1.6 Angle QMH equals the azimuth Z of the sun. Angle HMP equals the declination D of the wall. Angle QMP = angle MPS = angle W. We know the value of angle W from step E.1.5. For the diagram above and considering all angles positive, angle D = angle W - angle Z. To find angle D, we need the sun's azimuth Z at the exact time the shadow was marked.

Step 2. Find the sun's azimuth at the time of marking shadow.

E.2.0 If you have the program The Dialist's Companion, you can easily get the azimuth of the sun for the exact time the shadow was marked and skip to Step E.3.0. Otherwise, you can use the instructions which follow to compute the azimuth of the sun. Don't be surprised if your computed result differs slightly from that obtained from The Dialist's Companion, should you have a chance to compare.

E.2.1 To find the sun's azimuth, we need the Local Apparent Time LT for when the shadow was marked. If your timepiece showed Daylight Time, subtract one hour to get Standard Time ST. Next, you need your longitude (positive west, negative east) and your standard time meridian. In the United States, the Eastern Time zone has 75 degrees as its standard time meridian. Now, look up the value of the Equation of Time from the charts provided.

To convert from Standard Time ST to Local Apparent Time LT we need to deal with the correction for the longitudinal difference between your location and your standard time meridian, and the correction for the equation of time. Here's the formula:

$$LT = ST + (EQ + 4*(TM - LO)) / 60$$

where,

LT = local apparent time stated in decimal hours
ST = standard time converted to decimal hours
EQ = equation of time stated in decimal minutes
LO = the longitude of the place stated in decimal degrees
TM = the longitude of standard time meridian in decimal degrees
* means multiplied by
/ means divided by

EXAMPLE 1: You mark the plumb line's shadow at 10:09 (eastern) on 1/14/97 at longitude W 72.54 degrees. The standard time meridian is 75. The equation of time for 1/14/97 (from chart) is -9 min 7 sec. 10:09 is 10.15 decimal hours and -9 min 7 sec equals -9.12 minutes.

```
LT = 10.15 + (-9.12 + 4*(75 - 72.54)) / 60
   = 10.15 + (-9.12 + 9.84) / 60
   = 10.15 + 0.012
   = 10.16 hours              (about a half-minute difference)
```

EXAMPLE 2: You mark the shadow at 15:36 (3:36 pm) on 10/20/96 at longitude W 71.1. EQ is 15 min 14 sec. On 10/20/96 daylight time was still in effect, so subtract one hour. 14:36 is 14.6 hours and 15 min 14 sec equals 15.23 minutes.

```
LT = 14.6 + (15.23 + 4*(75 - 71.1)) / 60
   = 14.6 + (15.23 + 15.6) / 60
   = 14.6 + 0.514
```

```
   = 15.114 hours                   (or, 15 hr 6 min 50 sec)
```

EXAMPLE 3: ST is 13:55 on 2/24/97 at longitude 81.8. EQ is -13 min 14 sec. 13:55 is 13.917 hours and EQ converts to -13.233.min.

```
LT = 13.917 + (-13.233 + 4*(75 - 81.8)) / 60
   = 13.917 + (-13.233 + 4*(-6.8)) / 60
   = 13.917 + (-13.233 -27.2) / 60
   = 13.917 - 0.674
   = 13.243 hours                (or, 13 hr 14 min 35 sec)
```

E.2.2 To determine ST for any given LT, use this formula:

```
     ST = LT - (EQ + 4*(TM - LO)) / 60
```

EXAMPLE 4: To find ST for noon LT on 6-15-96 at longitude 80.3, let LT equal 12:00. EQ is -0 min 28 sec, which is -0.467 min.

```
ST = 12 - (-0.467 + 4*(75 - 80.3)) / 60
   = 12 - (-0.467 - 21.2) / 60
   = 12 - -0.36
   = 12.36  (remember to subtract algebraically)
   = 12:21:36   (this would be 13:21:36 Daylight Time)
```

EXAMPLE 5: To find ST for noon LT on 11-1-96 at longtitude 72.5. EQ is 16 min 27 sec, which is 16.45 min.

```
ST = 12 - (16.45 + 4*(75 - 72.5)) / 60
   = 12 - (16.45 + 10) / 60
   = 12 - 0.441
   = 11.559
   = 11:33:32
```

E.2.3 To compute the azimuth Z of the sun at the time the shadow was marked, we use the following formulas:

```
     N = Sin Lat * Cos HA - Cos Lat * Tan DC

     Tan Z = Sin HA / N

     (if N < 0,  then,  Z = Z + 180)
```

where,

N = a temporary value

DC = the declination of the sun (from chart)
HA = the hour angle of the sun from local noon at the instant of marking the plumb line's shadow, with hour angles before noon being negative and after noon, positive
Lat = the latitude of the place

EXAMPLE 6: You marked the plumb line's shadow at 10:09 (eastern) on 1/14/97 at longitude W 72.54 degrees. Let the latitude be N 42.97. In Example 1 above we computed LT as 10.16 hours. This is (12 - 10.16) or 1.84 hours before noon. Since the sun's apparent motion is 15 degrees per hour, 1.84 hours converts to 27.6 degrees for HA, the hour angle from local noon. We'll use -27.6 since the hour is before noon. The declination DC of the sun on Jan 14 was -21.25 degrees.

```
    N = Sin Lat * Cos HA - Cos Lat * Tan DC
      = 0.68161533 * 0.886203579 - 0.731710694 * -0.388878731
      = 0.604049945 - -0.284546727
      = 0.888596672

Tan Z = Sin HA / N
      = -0.463296035 / 0.888596672
      = -0.521298041
    Z = -27.53661001
```

We've computed the azimuth of the sun as -27.54 degrees (27.54s degrees east of south) at 10:09 on 1/14.

EXAMPLE 7: The shadow was marked at 15:36 (eastern daylight time) on 10/20/96 at longitude W 71.1, latitude N 32.7. LT then is 15.114, so HA equals 3.114 * 15 or 46.71. From the chart, DC = -10.44

```
    N = Sin Lat * Cos HA - Cos Lat * Tan DC
      = 0.54024032 * 0.685691322 - 0.841510781 * -0.184256085
      = 0.370438099 - -0.155053482
      = 0.525491581

Tan Z = Sin HA / N
      = 0.727892444 / 0.525491581
      = 1.385164805
    Z = 54.17313536
```

We've computed the azimuth of the sun as 54.17 degrees west of south at 15:36 DT on 10/20/96

Step 3. Use the data to determine the declination of the wall.

E.3.0 To compute the declination of the wall, follow these steps, which are adapted from page 93 of Waugh's book. For the purpose of sections 3.3.0 through 3.3.2, consider all angles as positive.

Z = the azimuth of the sun

W = the angle between the sun and a perpendicular to the wall

D = the declination of the wall

E.3.1 It's strongly recommended that you sketch an overhead view of the wall, its perpendicular, whether the sun was left or right of that perpendicular as you look straight out in the direction the wall faces, and whether the sun was east or west of south. (Note: the sun is due south exactly at noon, LT.) In Diagram #4, the sun is east of south and left of the wall's perpendicular. When using the rules in section 3.3.2 below, remember that your orientation is looking straight out in the direction the wall faces.

E.3.2 How to find D, the declination of the wall. Remember, in this section we are considering all angles as positive.

Case A - The time of the measurement was before noon, LT.

1) If the sun was right of the wall's perpendicular, then

$D = Z + W$ and the wall declines east.

2) If the sun was left of the wall's perpendicular, then,

if $Z > W$, then $D = Z - W$ and the wall declines east.

3) If the sun was left of the wall's perpendicular, then,

if $W > Z$, then $D = W - Z$ and the wall declines west.

Case B - The time of the measurement was after noon, LT.

1) If the sun was left of the wall's perpendicular, then

$D = Z + W$ and the wall declines west.

2) If the sun was right of the wall's perpendicular, then,

if $Z > W$, then $D = Z - W$ and the wall declines west.

3) If the sun was right of the wall's perpendicular, then,

if $W > Z$, then $D = W - Z$ and the wall declines east.

E.3.3 EXAMPLE 8: Let's apply the information of section 3.3.2.

Moore's Field is at latitude N 42.93 and longitude W 72.54. From a compass I knew the hanger wall faced approximately south, but an accurate declination was needed. I hung a plumb line 321 mm from a board placed against the wall and at 13:55:55 eastern time (13:56:31 Local Apparent Time) on 1/14/97 I marked the plumb line's shadow as 121 mm east of the point where a perpendicular from the plumb line met the board. This, of course, showed that the sun was west (or right) of the perpendicular to the wall. Remember, our orientation is looking out from the wall, not looking at the wall. From step E.1.5 above, the angle (W) between the sun and the wall's perpendicular was,

$$\tan W = 121 \text{ divided by } 321 = 0.37694704$$
$$W = 20.65 \text{ degrees}$$

The sun's azimuth for 13:55:55 on 1/14/97 (computed, or from The Dialist's Companion) was 28.98 degrees. Case B2 applies and the wall declines 8.3 degrees. Other readings were averaged in and our final determination was that the wall declines 8.4 degrees west of true south. That is the same as 188.4 degrees from north.

EXAMPLE 9:

Refer to DIAGRAM #3. At 11:57:55 eastern daylight time on 7-22-96 assume MP was 272 mm and MS was 640 mm.

$$\text{Angle } W = \text{ArcTan}\,(640 / 272) = 66.97 \text{ degrees.}$$

Refer to DIAGRAM #4. The azimuth Z of the sun was 32.96 degrees (east of south) at the time the shadow was marked. Case A3 applies here, for the sun was east (or left) of the wall's perpendicular and $W > Z$, so $D = W - Z$ and the wall declines 34.0 degrees west.

The following charts of Equation of Time and Solar Declination have been adapted, with permission, from The Dialist's Companion. The equation of time is presented as minutes and seconds. The sun's declination is given as decimal degrees.

Equation of Time and Solar Declination at Noon Standard Time
Standard Meridian 75W

January, Mean Year (1998)

	Equation	Declin		Equation	Declin		Equation	Declin
1	- 3:38	-22.97	11	- 7:59	-21.75	21	-11:22	-19.83
2	- 4:06	-22.89	12	- 8:22	-21.59	22	-11:38	-19.60
3	- 4:34	-22.79	13	- 8:45	-21.42	23	-11:54	-19.37
4	- 5:01	-22.69	14	- 9:07	-21.25	24	-12:09	-19.13
5	- 5:28	-22.57	15	- 9:28	-21.06	25	-12:23	-18.88
6	- 5:54	-22.46	16	- 9:49	-20.88	26	-12:36	-18.63
7	- 6:20	-22.33	17	-10:09	-20.68	27	-12:48	-18.38
8	- 6:46	-22.20	18	-10:28	-20.48	28	-13:00	-18.11
9	- 7:11	-22.06	19	-10:47	-20.27	29	-13:11	-17.85
10	- 7:35	-21.91	20	-11:05	-20.05	30	-13:21	-17.57
						31	-13:30	-17.29

February, Mean Year (1998)

	Equation	Declin		Equation	Declin		Equation	Declin
1	-13:38	-17.01	11	-14:17	-13.92	21	-13:38	-10.45
2	-13:46	-16.72	12	-14:16	-13.59	22	-13:31	-10.08
3	-13:52	-16.43	13	-14:15	-13.25	23	-13:23	- 9.72
4	-13:58	-16.13	14	-14:13	-12.91	24	-13:14	- 9.35
5	-14:03	-15.83	15	-14:10	-12.57	25	-13:04	- 8.98
6	-14:08	-15.52	16	-14:07	-12.22	26	-12:54	- 8.60
7	-14:11	-15.21	17	-14:02	-11.87	27	-12:44	- 8.23
8	-14:14	-14.89	18	-13:57	-11.52	28	-12:33	- 7.85
9	-14:15	-14.57	19	-13:52	-11.17			
10	-14:16	-14.25	20	-13:45	-10.81			

March, Mean Year (1998)

	Equation	Declin		Equation	Declin		Equation	Declin
1	-12:21	- 7.47	11	-10:00	- 3.60	21	- 7:09	+ 0.35
2	-12:09	- 7.09	12	- 9:44	- 3.20	22	- 6:51	+ 0.74
3	-11:56	- 6.71	13	- 9:28	- 2.81	23	- 6:33	+ 1.14
4	-11:43	- 6.32	14	- 9:11	- 2.42	24	- 6:15	+ 1.53
5	-11:30	- 5.94	15	- 8:54	- 2.02	25	- 5:57	+ 1.93
6	-11:16	- 5.55	16	- 8:37	- 1.63	26	- 5:39	+ 2.32
7	-11:01	- 5.16	17	- 8:20	- 1.23	27	- 5:21	+ 2.71
8	-10:47	- 4.77	18	- 8:03	- 0.84	28	- 5:03	+ 3.10
9	-10:32	- 4.38	19	- 7:45	- 0.44	29	- 4:45	+ 3.49
10	-10:16	- 3.99	20	- 7:27	- 0.05	30	- 4:27	+ 3.88
						31	- 4:09	+ 4.27

April, Mean Year (1998)

	Equation	Declin		Equation	Declin		Equation	Declin
1	- 3:51	+ 4.65	11	- 1:02	+ 8.42	21	+ 1:19	+11.96
2	- 3:33	+ 5.04	12	- 0:46	+ 8.79	22	+ 1:30	+12.29
3	- 3:15	+ 5.42	13	- 0:31	+ 9.15	23	+ 1:42	+12.63
4	- 2:58	+ 5.80	14	- 0:16	+ 9.51	24	+ 1:53	+12.96
5	- 2:41	+ 6.18	15	- 0:01	+ 9.87	25	+ 2:03	+13.28
6	- 2:24	+ 6.56	16	+ 0:13	+10.23	26	+ 2:13	+13.61
7	- 2:07	+ 6.94	17	+ 0:27	+10.58	27	+ 2:22	+13.93
8	- 1:50	+ 7.31	18	+ 0:40	+10.93	28	+ 2:31	+14.24
9	- 1:34	+ 7.68	19	+ 0:54	+11.27	29	+ 2:40	+14.55
10	- 1:17	+ 8.05	20	+ 1:06	+11.62	30	+ 2:48	+14.86

May, Mean Year (1998)

	Equation	Declin		Equation	Declin		Equation	Declin
1	+ 2:55	+15.16	11	+ 3:39	+17.95	21	+ 3:26	+20.24
2	+ 3:02	+15.46	12	+ 3:40	+18.21	22	+ 3:21	+20.44
3	+ 3:08	+15.76	13	+ 3:41	+18.45	23	+ 3:16	+20.63
4	+ 3:14	+16.05	14	+ 3:41	+18.70	24	+ 3:11	+20.82
5	+ 3:19	+16.34	15	+ 3:40	+18.93	25	+ 3:05	+21.00
6	+ 3:24	+16.62	16	+ 3:39	+19.17	26	+ 2:59	+21.17
7	+ 3:28	+16.89	17	+ 3:38	+19.39	27	+ 2:52	+21.34
8	+ 3:32	+17.17	18	+ 3:35	+19.61	28	+ 2:45	+21.51
9	+ 3:34	+17.43	19	+ 3:33	+19.83	29	+ 2:37	+21.66
10	+ 3:37	+17.70	20	+ 3:29	+20.04	30	+ 2:29	+21.81
						31	+ 2:20	+21.95

June, Mean Year (1998)

Day	Equation	Declin	Day	Equation	Declin	Day	Equation	Declin
1	+ 2:11	+22.09	11	+ 0:23	+23.10	21	- 1:45	+23.44
2	+ 2:02	+22.22	12	+ 0:10	+23.17	22	- 1:58	+23.43
3	+ 1:52	+22.35	13	- 0:02	+23.22	23	- 2:11	+23.42
4	+ 1:42	+22.46	14	- 0:15	+23.27	24	- 2:24	+23.40
5	+ 1:31	+22.58	15	- 0:28	+23.32	25	- 2:37	+23.38
6	+ 1:21	+22.68	16	- 0:40	+23.36	26	- 2:50	+23.35
7	+ 1:10	+22.78	17	- 0:53	+23.39	27	- 3:02	+23.31
8	+ 0:58	+22.87	18	- 1:06	+23.41	28	- 3:14	+23.26
9	+ 0:47	+22.95	19	- 1:19	+23.42	29	- 3:27	+23.21
10	+ 0:35	+23.03	20	- 1:32	+23.43	30	- 3:38	+23.15

July, Mean Year (1998)

Day	Equation	Declin	Day	Equation	Declin	Day	Equation	Declin
1	- 3:50	+23.09	11	- 5:29	+22.06	21	- 6:23	+20.41
2	- 4:01	+23.01	12	- 5:37	+21.93	22	- 6:26	+20.21
3	- 4:13	+22.93	13	- 5:44	+21.78	23	- 6:28	+20.01
4	- 4:23	+22.85	14	- 5:51	+21.63	24	- 6:29	+19.80
5	- 4:34	+22.76	15	- 5:57	+21.48	25	- 6:30	+19.59
6	- 4:44	+22.66	16	- 6:03	+21.31	26	- 6:30	+19.37
7	- 4:54	+22.55	17	- 6:08	+21.14	27	- 6:29	+19.14
8	- 5:03	+22.44	18	- 6:13	+20.97	28	- 6:28	+18.91
9	- 5:12	+22.32	19	- 6:17	+20.79	29	- 6:27	+18.68
10	- 5:21	+22.19	20	- 6:20	+20.60	30	- 6:24	+18.44
						31	- 6:22	+18.19

August, Mean Year (1998)

Day	Equation	Declin	Day	Equation	Declin	Day	Equation	Declin
1	- 6:18	+17.94	11	- 5:10	+15.18	21	- 3:07	+12.01
2	- 6:14	+17.68	12	- 5:00	+14.88	22	- 2:52	+11.68
3	- 6:09	+17.42	13	- 4:50	+14.58	23	- 2:36	+11.34
4	- 6:04	+17.16	14	- 4:39	+14.27	24	- 2:20	+11.00
5	- 5:58	+16.89	15	- 4:27	+13.96	25	- 2:04	+10.65
6	- 5:52	+16.62	16	- 4:15	+13.64	26	- 1:47	+10.31
7	- 5:45	+16.34	17	- 4:02	+13.32	27	- 1:30	+ 9.96
8	- 5:37	+16.05	18	- 3:49	+13.00	28	- 1:13	+ 9.61
9	- 5:29	+15.77	19	- 3:36	+12.68	29	- 0:55	+ 9.25
10	- 5:20	+15.47	20	- 3:21	+12.35	30	- 0:36	+ 8.89
						31	- 0:18	+ 8.53

September, Mean Year (1998)

	Equation	Declin		Equation	Declin		Equation	Declin
1	+ 0:01	+ 8.17	11	+ 3:22	+ 4.44	21	+ 6:55	+ 0.59
2	+ 0:20	+ 7.81	12	+ 3:44	+ 4.06	22	+ 7:17	+ 0.20
3	+ 0:39	+ 7.44	13	+ 4:05	+ 3.68	23	+ 7:38	- 0.19
4	+ 0:59	+ 7.07	14	+ 4:26	+ 3.30	24	+ 7:59	- 0.58
5	+ 1:19	+ 6.70	15	+ 4:47	+ 2.91	25	+ 8:19	- 0.97
6	+ 1:39	+ 6.33	16	+ 5:09	+ 2.53	26	+ 8:40	- 1.36
7	+ 1:59	+ 5.95	17	+ 5:30	+ 2.14	27	+ 9:00	- 1.75
8	+ 2:20	+ 5.58	18	+ 5:51	+ 1.75	28	+ 9:21	- 2.14
9	+ 2:41	+ 5.20	19	+ 6:13	+ 1.36	29	+ 9:41	- 2.53
10	+ 3:01	+ 4.82	20	+ 6:34	+ 0.98	30	+10:01	- 2.91

October, Mean Year (1998)

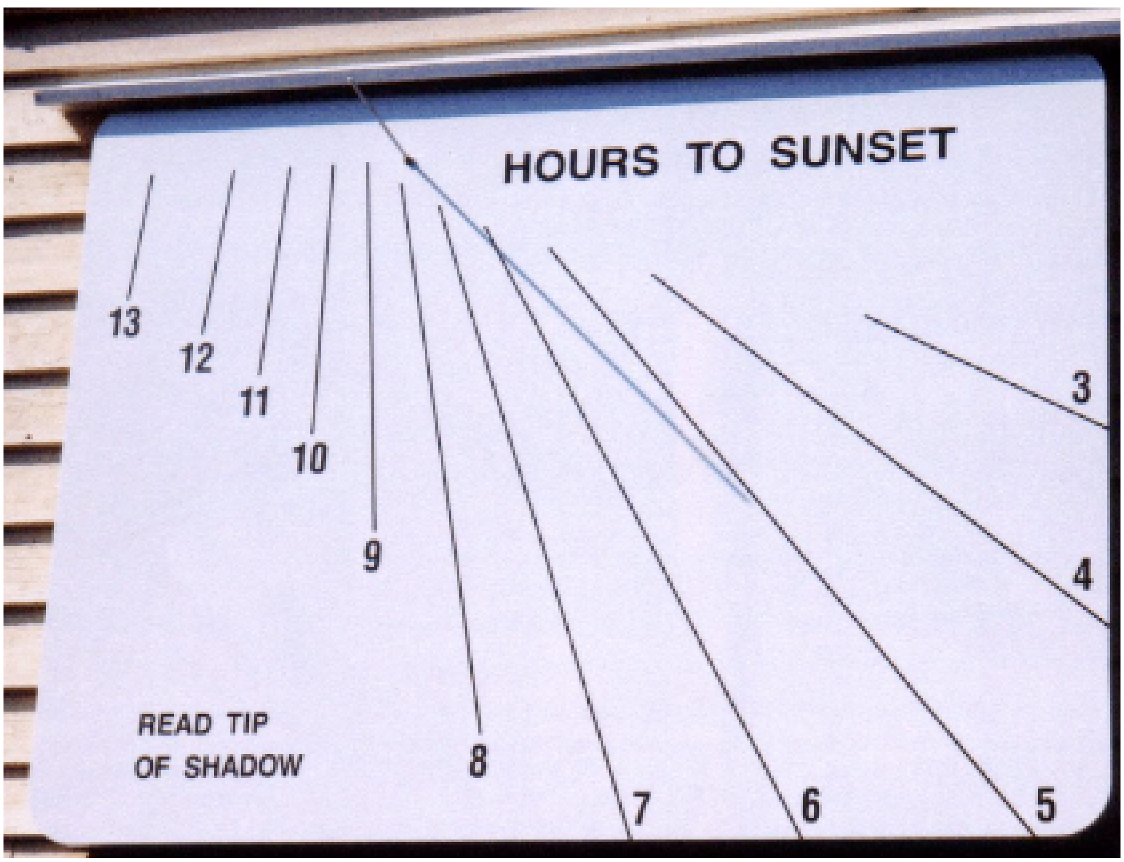
	Equation	Declin		Equation	Declin		Equation	Declin
1	+10:20	- 3.30	11	+13:16	- 7.13	21	+15:24	-10.80
2	+10:39	- 3.69	12	+13:32	- 7.51	22	+15:33	-11.15
3	+10:58	- 4.08	13	+13:46	- 7.88	23	+15:42	-11.50
4	+11:17	- 4.46	14	+14:01	- 8.25	24	+15:50	-11.85
5	+11:35	- 4.85	15	+14:14	- 8.62	25	+15:57	-12.19
6	+11:53	- 5.23	16	+14:27	- 8.99	26	+16:04	-12.54
7	+12:11	- 5.61	17	+14:40	- 9.36	27	+16:09	-12.87
8	+12:28	- 5.99	18	+14:52	- 9.72	28	+16:14	-13.21
9	+12:44	- 6.37	19	+15:03	-10.08	29	+16:19	-13.54
10	+13:01	- 6.75	20	+15:14	-10.44	30	+16:22	-13.87
						31	+16:25	-14.20

November, Mean Year (1998)

	Equation	Declin		Equation	Declin		Equation	Declin
1	+16:27	-14.52	11	+15:59	-17.50	21	+14:07	-19.97
2	+16:28	-14.83	12	+15:52	-17.77	22	+13:51	-20.19
3	+16:28	-15.15	13	+15:44	-18.03	23	+13:35	-20.40
4	+16:27	-15.46	14	+15:34	-18.30	24	+13:17	-20.60
5	+16:26	-15.76	15	+15:24	-18.55	25	+12:59	-20.80
6	+16:24	-16.06	16	+15:14	-18.80	26	+12:40	-20.99
7	+16:20	-16.36	17	+15:02	-19.05	27	+12:21	-21.18
8	+16:16	-16.65	18	+14:49	-19.29	28	+12:00	-21.35
9	+16:12	-16.94	19	+14:36	-19.52	29	+11:39	-21.52
10	+16:06	-17.22	20	+14:22	-19.75	30	+11:18	-21.69

December, Mean Year (1998)

	Equation	Declin		Equation	Declin		Equation	Declin
1	+10:55	-21.84	11	+ 6:41	-23.02	21	+ 1:52	-23.44
2	+10:32	-21.99	12	+ 6:14	-23.10	22	+ 1:22	-23.44
3	+10:09	-22.14	13	+ 5:45	-23.16	23	+ 0:52	-23.43
4	+ 9:45	-22.27	14	+ 5:17	-23.23	24	+ 0:22	-23.41
5	+ 9:20	-22.40	15	+ 4:48	-23.28	25	- 0:07	-23.39
6	+ 8:55	-22.52	16	+ 4:19	-23.32	26	- 0:37	-23.35
7	+ 8:29	-22.64	17	+ 3:50	-23.36	27	- 1:07	-23.31
8	+ 8:03	-22.74	18	+ 3:20	-23.39	28	- 1:36	-23.26
9	+ 7:36	-22.84	19	+ 2:51	-23.42	29	- 2:05	-23.21
10	+ 7:09	-22.93	20	+ 2:21	-23.43	30	- 2:34	-23.14
						31	- 3:03	-23.07



This vertical declining dial was designed for 42.85 N, 72.55 W, and faces 46.56 degrees east of south. It shows just over 5 hours until sunset. The painted signboard is about 33 by 45 inches, and has viny letters, numerals, and hour lines. As an amusement, I later added a large brass-colored winding key to the dial face.