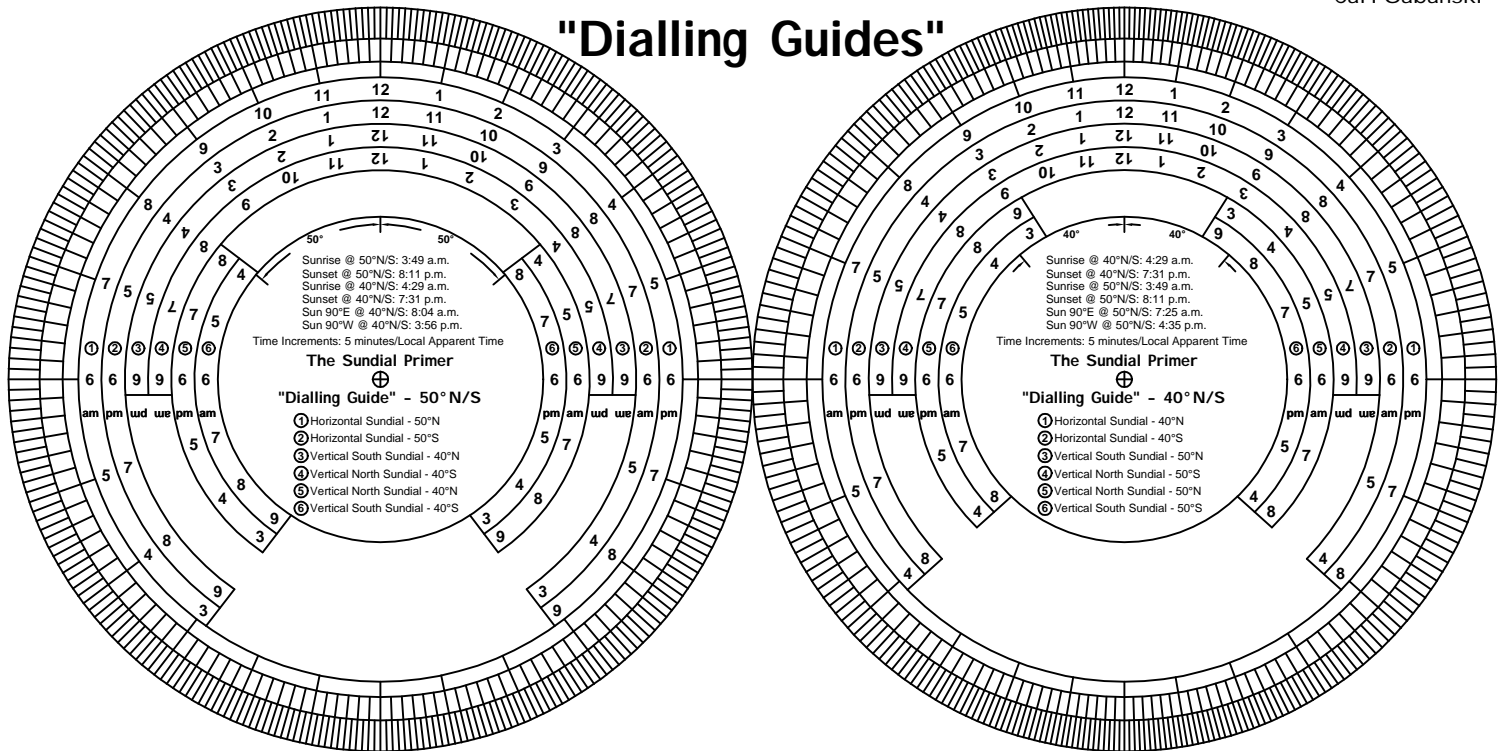


The Sundial Primer

"Dialling Guides"

created by
Carl Sabanski



The purpose of the "Dialling Guides" is to provide an easy method for laying out the hour lines for a number of horizontal and vertical sundials located at various latitudes in either the Northern or Southern Hemispheres. There are a total of 81 "Dialling Guides" that cover the range of latitudes from 25° to 65° inclusive in increments of 0.5°. Each "Dialling Guide" illustrates how to lay out the hour lines for the following sundials in minimum time increments of 5 minutes:

1. Horizontal Sundial - Northern Hemisphere
2. Horizontal Sundial - Southern Hemisphere
3. Vertical Direct South Sundial - Northern Hemisphere
4. Vertical Direct North Sundial - Southern Hemisphere
5. Vertical Direct North Sundial - Northern Hemisphere
6. Vertical Direct South Sundial - Southern Hemisphere

The sundials are numbered on the "Dialling Guides" as indicated above.

For any given latitude the "Dialling Guides" can be used to design the following sundials:

1. Horizontal Sundial
2. Vertical Direct South Sundial
3. Vertical Direct North Sundial

In order to design these three sundials for a given latitude, two "Dialling Guides" must be obtained. The "Dialling Guide" for the desired latitude is required for the horizontal sundial and one for the co-latitude is required for the vertical direct south and north sundials. Note that a vertical direct south sundial at any latitude in the Northern Hemisphere is equivalent to a horizontal sundial at the co-latitude in the Southern Hemisphere. A vertical direct north sundial in the Southern Hemisphere is equivalent to a horizontal sundial in the Northern Hemisphere. Finding an equivalent horizontal sundial for a given vertical sundial is called "reduction". The vertical direct north sundial in the Northern Hemisphere is an inverted mirrored image of the vertical direct south sundial. The relationship between the vertical direct south and north sundials in the Southern Hemisphere is the same.

In order to set the limits of the hour lines for the individual sundials certain parameters must be known. These are listed on each individual "Dialling Guide" and are as follows:

1. Sunrise: The earliest time of sunrise at the given latitude. This occurs on the Summer Solstice.
2. Sunset: The latest time of sunset at any given latitude. This occurs on the Summer Solstice.
3. Sun 90°E: The latest time the sun reaches due east at the given latitude. At this time the sun will stop illuminating vertical north (Northern Hemisphere) and south (Southern Hemisphere) sundials and start illuminating vertical south and north sundials in the respective hemispheres. This occurs on the Summer Solstice.
4. Sun 90°W: The earliest time the sun reaches due west at the given latitude. At this time the sun will stop illuminating vertical south (Northern Hemisphere) and north (Southern Hemisphere) sundials and start illuminating vertical north and south sundials in the respective hemispheres. This occurs on the Summer Solstice.

These parameters are used to determine the limits of the hour lines for the various sundials. The times listed are approximate but adequate for establishing the hour limits. For the design of a sundial at latitude "LAT", these limits are determined as follows:

1. Horizontal Sundial: The sundial will display the hours from the earliest sunrise, "Sunrise", to the latest sunset, "Sunset" for the desired latitude "LAT".
2. Vertical South (Northern Hemisphere) and Vertical North (Southern Hemisphere): The sundials will display the hours from 6 a.m. to 6 p.m. The hour limits are set at the Equinoxes. The sundials can display a maximum of 12 hours.
3. Vertical North (Northern Hemisphere) and Vertical South (Southern Hemisphere) Sundials: For the range of latitudes chosen for the "Dialling Guides" the sundials will display a number of hours in the morning and then again in the afternoon. The morning hour limits are from the earliest morning sunrise, "Sunrise", to the latest time the sun will appear due east, "Sun 90°E". The afternoon hours are from the earliest time the sun will appear due west, "Sun 90°W", to the latest time of sunset, "Sunset". The values are determined at latitude "LAT".

It is interesting to note that once the design of a vertical north (Northern Hemisphere) or vertical south (Southern Hemisphere) sundial moves beyond the Arctic/Antarctic Circles the sundials will actually be illuminated by the sun for periods of time greater than 12 hours. Make a "Dialling Buddy" and it will show all this and more. It is a very interesting device!

Each "Dialling Guide" also has two short lines that are rotated at an angle equal to the latitude from the vertical 12 line. The lines are used to draw the style of the gnomon at the correct angle. The "style height" of a horizontal sundial designed for a given latitude "LAT" is the angle the style makes with the horizontal dial plate and is equal to "LAT". The "style height" of the vertical sundials designed for a given latitude "LAT" is the angle the style makes with the vertical dial plate and is equal to the co-latitude, 90° minus "LAT".

There are a few other items to note regarding the "Dialling Guides":

1. The hour lines have been numbered in a way that orientates the "Dialling Guide" correctly for the sundial being designed. That is to say, the hour numbers for the vertical south (Northern Hemisphere) and vertical north (Southern Hemisphere) sundials are inverted so that when the "Dialling Guide" is rotated 180° the sundials appear as they would on a wall.
2. The hour number 12 for the vertical north (Northern Hemisphere) and vertical south (Southern Hemisphere) sundials is midnight. For all the other sundials the hour number 12 is noon.
3. All the "Dialling Guides" have been designed to show local apparent or solar time.
4. The hour limits are rounded off to the nearest full hour that includes the appropriate parameter for the particular sundial. Note the actual hour limits and adjust the sundial hour lines accordingly.
5. If a "Dialling Guide" is printed at full scale it can be used to draw a sundial on a standard letter size sheet of paper. Be careful not to reduce the printout too much as it may become difficult to distinguish the hour lines.

If you have obtained copies of the "Dialling Guides" for your latitude and co-latitude you are almost ready to start laying out your sundial. The one thing you must decide is how wide your gnomon will be. The width of the gnomon and how you accommodate it will affect the accuracy of your sundial. There is nothing wrong with a wide gnomon and in many designs a wide gnomon is necessary so that it can be mounted securely. It is quite easy to adjust the hour line layout for a wide gnomon. If you choose not to do so the accuracy of the sundial will be reduced because with a wide gnomon the shadow casting edge changes 3 times a day, at 6 a.m., noon and 6 p.m.. The affect is more pronounced on small sundials as the hour lines are closer together.

If the width of the gnomon is not a consideration then the sundial will have a single centre or origin as shown in Figure 1. Draw a horizontal line AB and a vertical line CD through the centre of AB. Both these lines should be longer than the diameter of the "Dialling Guide". The centre of the sundial is O and all the hour lines originate from this point. Place the "Dialling Guide" with its 6 a.m. and 6 p.m. lines on the line AB and the 12 line on the line CD. The centre of the "Dialling Guide" will now coincide with the centre O of the sundial. Mark off the hour lines you wish to display on your sundial. Remove the "Dialling Guide" and complete the design of your sundial. Rotate the "Dialling Guide" 180° if you are designing sundials 3 or 4.

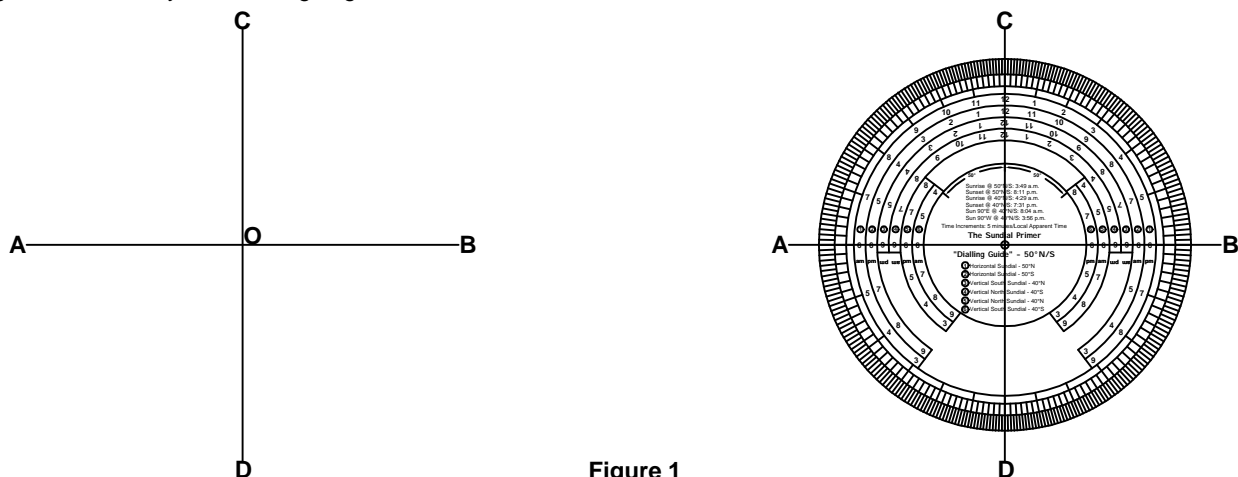


Figure 1

If you have a wide gnomon and choose to compensate for it, this is not very difficult. In this case the sundial will have two centres or origins as shown in Figure 2. Assume that the width of the gnomon is W . Draw a horizontal line AB and a vertical line CD through the centre of AB . Both these lines should be longer than the diameter of the "Dialling Guide" plus the width of the gnomon. Draw a line EF parallel and to the left of CD at a distance equal to half the width of the gnomon or $W/2$. Draw a second line GH parallel and to the right of CD at a distance equal to half the width of the gnomon or $W/2$. Lines EF and GH cross the horizontal line AB at X and Y . These are now the two centres of the sundial and the hour lines will originate from one of these two points. The point from which a particular hour line will originate will depend upon the sundial and the time of day. The "Dialling Guide" is aligned as described previously but instead of using the vertical line CD , the line EF or GH is used.

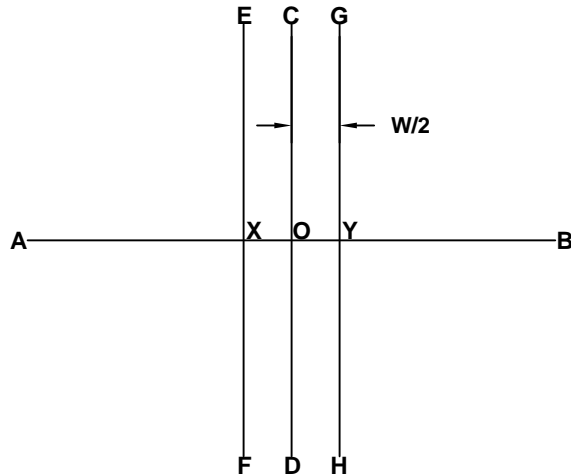
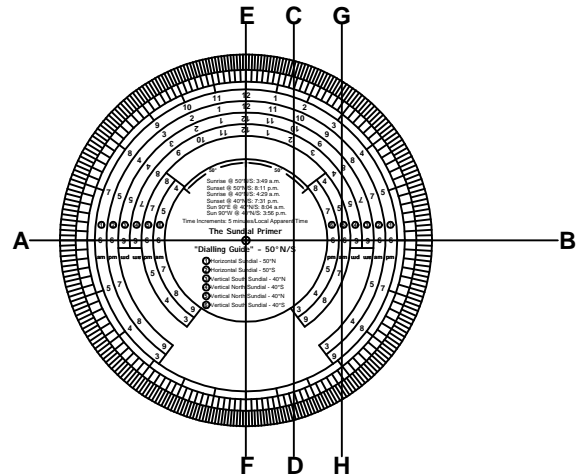


Figure 2



The following describes the hour lines that originate from the two centres of the sundial X and Y for the various sundials listed on the "Dialling Guide".

1. Horizontal Sundial - Northern Hemisphere

X : hour lines between 6 a.m. and 12 noon; hour lines after 6 p.m.
 Y : hour lines before 6 a.m.; hour lines between 12 noon and 6 p.m.

2. Horizontal Sundial - Southern Hemisphere

X : hour lines before 6 a.m.; hour lines between 12 noon and 6 p.m.
 Y : hour lines between 6 a.m. and 12 noon; hour lines after 6 p.m.

3. Vertical South Sundial - Northern Hemisphere

X : hour lines between noon and 6 p.m.
 Y : hour lines between 6 a.m. and noon

4. Vertical North Sundial - Southern Hemisphere

X : hour lines between 6 a.m. and noon
 Y : hour lines between noon and 6 p.m.

5. Vertical North Sundial - Northern Hemisphere

X : hour lines after 6 a.m.; hour lines between 6 p.m. and midnight
 Y : hour lines between midnight and 6 a.m.; hour lines before 6 p.m.

6. Vertical South Sundial - Southern Hemisphere

X : hour lines between midnight and 6 a.m.; hour lines before 6 p.m.
 Y : hour lines after 6 a.m.; hour lines between 6 p.m. and midnight

A sundial with a wide gnomon will have 2 noon hour lines.

The space between these two lines is called the "noon gap".

A horizontal sundial with a wide gnomon located beyond the Arctic/Antarctic Circles will have 2 midnight hour lines.

The hour lines between the 2 midnight lines actually overlap. Let's call this a "midnight overlap".

There is a type of gnomon that can change the gaps to overlaps and the overlaps to gaps.

For more information visit The Sundial Primer page "The Wide Gnomon" where you will find a number of images that illustrate the shadow moving from one edge of the gnomon to another. There are also illustrations of completed layouts.

During the design of the sundial consideration must be given to the dial plate. What shape will it be? How large will it be? How long will the hour lines be? Where will the hour line numbers be placed? Once these questions and others have been answered you will have a dial plate design. One other item that needs to be considered is the gnomon. Its size and shape are very important. The typical gnomon shape forms a triangle and that is what will be discussed here. The critical factor is the size and position of the style. Once that is determined the remaining design of the gnomon can be left to your imagination. Figure 3 shows simple gnomons for a horizontal and vertical sundial.

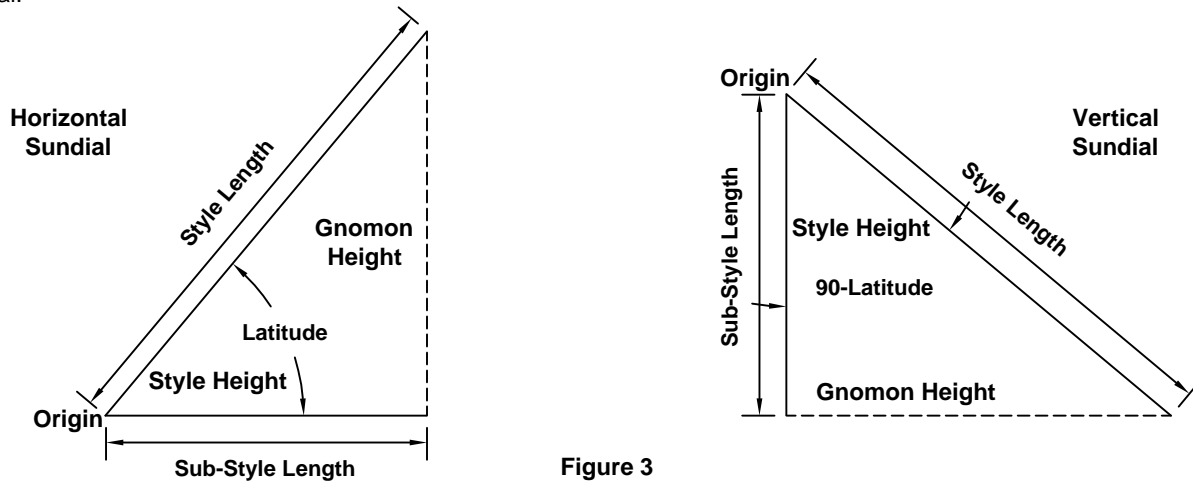


Figure 3

The style is the part of the gnomon that casts the shadow used to indicate the time on the dial plate. The origin is the point on the dial plate from which all the hour lines radiate. As discussed earlier, a wide gnomon will have two styles and as a result two origins. The style height, an angular measurement, for a horizontal sundial is the angle of the style relative to the horizontal dial plate and is equal to the latitude. For a vertical sundial the style height is the angle of the style relative to the vertical dial plate and is equal to the co-latitude. What now must be determined is the required style length.

The factor that determines this measurement is how long must the shadow be to make the sundial readable throughout the entire year. Where do you want the shadow to be when it is the shortest? The shadow of a horizontal sundial is shortest at solar noon on the summer solstice and for a vertical sundial this occurs at solar noon on the winter solstice.

Figure 4 illustrates a horizontal sundial with the sun at the summer solstice position. The minimum shadow length cast by the gnomon at solar noon on this day is calculated as follows:

Calculate the sun's maximum altitude: $AMAX = (90 - \text{Latitude} + 23.44)^\circ$
 Minimum Shadow Length = Gnomon Height / $\tan(AMAX)$

The style length required to achieve the desired shadow length for your design is calculated as follows:

Style Length = $1.09 * (\text{Sub-Style Length} + \text{Shadow Length}) * \sin(113.44 - \text{Latitude})$

This calculation is made simple by using Table 1. The table is based on "Gnomon Height" and "Sub-Style Length + Shadow Length" values of 1. The values of the "Minimum Shadow Length" (SMIN) and "Style Length" are given for the range of latitudes from 25° to 65° inclusive in increments of 0.5°. To calculate the actual "Minimum Shadow Length" multiply the actual "Gnomon Height" by the "SMIN" value obtained from the table for your latitude. To calculate your actual "Style Length" multiply the required "Sub-Style Length + Shadow Length" by the value obtained from the table for your latitude. The units don't matter and can be centimetres, inches, metres, feet or whatever.

Figure 5 illustrates a vertical sundial with the sun at the winter solstice position. The minimum shadow length cast by the gnomon at solar noon on this day is calculated as follows:

Calculate the sun's minimum altitude: $AMIN = (90 - \text{Latitude} - 23.44)^\circ$
 Shadow Length = Gnomon Height / $\tan(90 - AMIN)$

The style length required to achieve the desired shadow length for your design is calculated as follows:

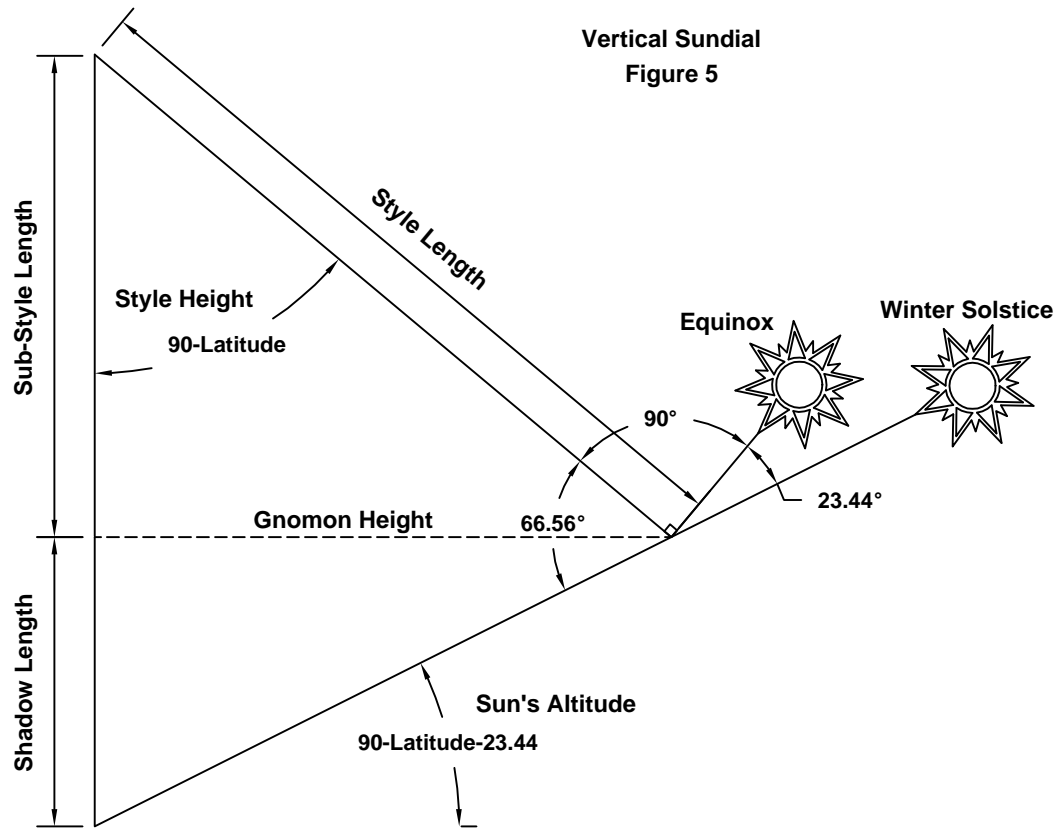
Style Length = $1.09 * (\text{Base Length} + \text{Shadow Length}) * \sin(\text{Latitude} + 23.44)$

This calculation is made simple by using Table 2. The information and procedures provided above for Table 1 also apply to Table 2.

Happy Dialling!

The Sundial Primer - "Dialling Guides"

Horizontal & Vertical Sundials



LATITUDE DEGREES	SMIN	STYLE LENGTH	LATITUDE DEGREES	SMIN	STYLE LENGTH	LATITUDE DEGREES	SMIN	STYLE LENGTH	LATITUDE DEGREES	SMIN	STYLE LENGTH
25	0.887	0.816	35.5	0.602	0.934	45.5	0.385	1.017	55.5	0.195	1.070
25.5	0.871	0.822	36	0.590	0.939	46	0.375	1.021	56	0.186	1.072
26	0.856	0.828	36.5	0.579	0.943	46.5	0.365	1.024	56.5	0.177	1.073
26.5	0.841	0.834	37	0.567	0.948	47	0.355	1.027	57	0.168	1.075
27	0.826	0.840	37.5	0.556	0.953	47.5	0.345	1.030	57.5	0.159	1.076
27.5	0.812	0.846	38	0.544	0.957	48	0.336	1.033	58	0.151	1.078
28	0.797	0.852	38.5	0.533	0.962	48.5	0.326	1.036	58.5	0.142	1.079
28.5	0.783	0.858	39	0.522	0.966	49	0.316	1.039	59	0.133	1.081
29	0.769	0.864	39.5	0.511	0.971	49.5	0.307	1.042	59.5	0.124	1.082
29.5	0.755	0.870	40	0.500	0.975	50	0.297	1.045	60	0.115	1.083
30	0.742	0.876	40.5	0.489	0.979	50.5	0.288	1.047	60.5	0.106	1.084
30.5	0.728	0.881	41	0.478	0.983	51	0.278	1.050	61	0.097	1.085
31	0.715	0.887	41.5	0.468	0.987	51.5	0.269	1.053	61.5	0.089	1.086
31.5	0.702	0.892	42	0.457	0.991	52	0.260	1.055	62	0.080	1.087
32	0.689	0.898	42.5	0.446	0.995	52.5	0.250	1.057	62.5	0.071	1.087
32.5	0.676	0.903	43	0.436	0.999	53	0.241	1.060	63	0.062	1.088
33	0.663	0.908	43.5	0.426	1.003	53.5	0.232	1.062	63.5	0.053	1.088
33.5	0.651	0.914	44	0.415	1.007	54	0.223	1.064	64	0.045	1.089
34	0.639	0.919	44.5	0.405	1.010	54.5	0.214	1.066	64.5	0.036	1.089
34.5	0.626	0.924	45	0.395	1.014	55	0.205	1.068	65	0.027	1.090
35	0.614	0.929									

Table 2